

ESSAYS ON NOMINAL GDP TARGETING

by

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## DISSERTATION ABSTRACT

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Title: Essays on Nominal GDP Targeting

The subject of this dissertation is nominal GDP (NGDP) targeting. In the wake of the Great Recession some economists have proposed using some form of NGDP target to replace current monetary policy. We evaluate the desirability of NGDP targets based upon their ability to deliver unique and “learnable” equilibria and their welfare gains in the presence of financial frictions.

In the second chapter we assess the determinacy and E-stability conditions for simple interest rate rules which respond to NGDP’s deviation from target in a simple three equation New Keynesian model. The rules under consideration target either NGDP level or growth, and can either be contemporaneous, one period ahead, or two periods ahead. We also allow for different types of information sets for the agents.

In the third chapter we compare welfare loss in consumption equivalent terms for NGDP targets with more conventional monetary policy in a New Keynesian model which features financial frictions.

Finally in the fourth chapter we continue our analysis from chapter one but now allow for strictly positive trend inflation. We present findings for the relationship between trend inflation and the determinacy and E-stability of the equilibrium when using interest rate rules that target NGPD.

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## CHAPTER I

### INTRODUCTION

Nominal GDP (NGDP) targeting has been nominated as an alternate intermediate target for monetary policy. Some of the reasons for its nomination are that it weights inflation and output stabilization equally, its desirable zero lower bound properties, and the countercyclical movements in real debt it creates.

In this dissertation we analyze the desirability of these policies along three different dimensions. Using a simple two equation New Keynesian model we examine whether interest rate rules which adjust to deviations from target for NGDP growth or level will lead to a determinate steady state equilibrium under rational expectations and whether that rational expectations equilibrium (REE) is learnable. Then we look at a New Keynesian model which now features financial frictions and heterogeneous preferences and examine whether welfare is higher under NGDP based monetary policy or other traditional monetary policies. Finally we return to the questions of determinacy and E-stability but allow for non-zero trend inflation and partial indexation to past and/or trend inflation.

In Chapter 2 we look at interest rate feedback rules that respond to deviations of nominal GDP growth away from the central bank's targeted value. In particular we examine the determinacy and E-stability properties of these rules. We find that these determinacy and E-stability properties vary based upon whether the rule responds to current or expected NGDP growth. We do find however that an interest rate rule that responds strictly positively with deviations of current NGDP level from its target will yield both determinacy and E-stability.

In Chapter 3 we use a New Keynesian model with financial frictions like the one developed in Curdia and Woodford (2016) and we compare the welfare

performance of four different NGDP targeting policies with the optimal policy monetary policy and different traditional policies. We find that most of the NGDP based policies provide higher welfare than traditional policies.

In Chapter 4 we return to a simple New Keynesian model but now allow the steady-state gross inflation to not necessarily equal one. We also assume that prices may not be fully indexed to past or trend inflation. It has been shown that these assumptions lead to changes in the determinacy and E-stability properties for a typical Taylor rule. We analyze the determinacy and E-stability properties of different NGDP targeting policies. We somewhat surprisingly find that it requires fairly high levels of trend inflation before the determinacy and E-stability properties of these interest rate rules change.

## CHAPTER II

### NGDP TARGETING, FEEDBACK RULES, DETERMINACY, AND E-STABILITY

#### 2.1 Introduction

The focus of this paper is the stability both under rational expectations and adaptive learning of equilibria of models with “Taylor type” interest rate rules that target nominal GDP (NGDP). For these rules NGDP’s growth rate or level is the variable the interest rate responds to rather than having separate response parameters for inflation and the output gap.

There is evidence to suggest that targeting NGDP has some desirable qualities (Hall and Mankiw, 1994; Taylor, 1985; McCallum and Nelson, 1999a). Part of the appeal is having one metric indicating how close a central bank is to its goals, rather than having separate measures linked to employment and prices. By the quantity theory of money, provided the velocity of money is exogenous, NGDP is determined directly by the money supply. As a result NGDP ends up being a natural candidate for an intermediate target for the central bank to pursue. Furthermore its movements are correlated with the outcome variables that central banks are typically most concerned with, such as the dual mandate of price stability and full employment in the United States. With nominal GDP as the intermediate target when real output falls pushing nominal output below target, there isn’t a separate inflation target to worry about that ties policymakers hands when trying to respond to the fall in real output.

For nominal GDP level targeting there are advantages that come with the history dependence of levels targeting. When shocks cause the economy to miss

target, policymakers make up for it. Woodford (2012) points out that this feature along with other features make it similar to the optimal policy at the zero lower bound from Eggertsson and Woodford (2003).

From a practical policy perspective nominal GDP targeting has drawn the interest of actual policy-makers as well. Writing about the issue of the long-term downward trend in the natural rate of interest Williams (2016) discusses the advantages of nominal GDP targeting as a policy alternative when the natural rate of interest is very close to zero.

Another advocate for the policy, Sumner (2012), offers the following as practical anecdote in favor:

When Bernanke calls for higher inflation, he means a higher level of aggregate demand, which economic theory suggests should raise both the inflation rate and (in the short run) the real incomes of Americans. In contrast, when average Americans hear the term higher inflation, they think in terms of a reduction in aggregate supply (perhaps higher food and energy prices), which reduces the real incomes of average Americans.

The moral of the story is that the public at large may not understand what inflation is, particularly the “inflation” that central banks target. Nominal GDP however is an easy target for the public to understand, thus making nominal GDP a good candidate for a nominal anchor because it won’t lead to outside pressure to engage in contractionary policy when real output is low while inflation is above target.

This paper looks at two general types of feedback rules in a New Keynesian framework and assesses the uniqueness of the steady-state equilibrium in the

model, also known as determinacy, and whether the equilibrium is expectationally-stable (E-stable), meaning that agents in this economy could learn the structural relationships between variables in the rational expectations equilibrium (REE). Uniqueness is of concern because indeterminacy of the steady state implies that equilibria not determined by fundamentals of the model, sunspot equilibria, may arise when the interest rate rules are implemented in the model. The learnability of the REE is important because rational expectations supposes that agents always know the structural characteristics of the economy in the model as well as the distribution of the exogenous shocks. An assumption closer to reality would be that agents don't know those structural parameters with certainty but they are estimating the relationships between endogenous and exogenous variables like an econometrician might. We want to know whether the REE associated with a simple linearized New Keynesian model and the class of rules we look at results in E-stable rational expectations equilibria or if the degree to which the central bank responds to deviations in nominal GDP growth affects the E-stability of the REE. The two types of interest rate rules under consideration are ones which target either the growth or level of nominal GDP. We consider various timing and information assumptions as well.

Using feedback rules is in contrast to a lot of the nominal GDP targeting literature Mitra (2003); Garín et al. (2016); Sheedy (2014); McCallum (2015); Honkapohja and Mitra (2014) which assumes that interest rates are set so as to precisely achieve a targeted level or growth rate of nominal GDP. This assumption however may grant too much knowledge of the structural parameters of the economy to the central bank. Actual central bank behavior in pursuing nominal GDP targeting as a policy goal may be better approximated by a rule of thumb for adjusting the policy instrument based upon the growth rate of nominal GDP. When



nominal GDP grows above its target, the rule increases the interest rate which in this model decreases the output gap and simultaneously decreases inflation, which will push growth rate towards the targeted value of nominal GDP.

For these types of rules we have reason to doubt that they will always result in determinate and E-Stable equilibria because there are determinacy and E-stability conditions on parameters of the Taylor rule and model parameters that arise from more standard Taylor rules in New Keynesian models as demonstrated by Bullard and Mitra (2002).

Our biggest results are: i.) When targeting current NGDP growth, regardless of information assumptions, a response of greater than one for one to deviations of NGDP growth from its target ensures both determinacy and E-stability. ii.) When targeting current NGDP level regardless of information assumptions, any strictly positive response to deviations from target will result in both determinacy and E-stability. iii.) We find analytic results for determinacy and E-stability conditions for the one period ahead expected NGDP growth interest rate rule. These results indicate that certain parameterizations can trigger what is called "inverted aggregate demand logic" by Bilbiie (2008), a situation where the response of output to the real interest rate becomes positive.

In Section 2 we provide an overview of the literature concerning Nominal GDP targeting in general and then learning and monetary policy with a specific focus on learning and nominal GDP targeting. In Section 3 we layout the model and define the different interest rate rules. In Section 4 we present the determinacy and E-stability results associated with these rules under the different information assumptions.

## 2.2 Literature Review

We build on two different strands of literature - measuring the desirability of nominal GDP targeting and evaluating monetary policy in the context of learning.

### Nominal GDP Targeting

Hall and Mankiw (1994) provides justification for nominal GDP targeting based upon four criteria for monetary policy rules that they set out - *efficiency*, *simplicity*, *precision*, and *accountability*. They argue that nominal GDP targeting as a policy rule hits all four of these desirability criteria. Hall and Mankiw point out that it is a policy that can supported by both the Real Business Cycle and New Keynesian schools of thought because in an RBC framework nominal GDP targeting will lead to price stability and in an NK framework nominal GDP targeting is a compromise on the response to a shock to the price level, reducing output by the amount the price level increased rather than contracting output until the price level goes back to target.

The authors assume a central bank capable of setting the interest rate to hit its target either perfectly or with white noise error. They discuss the merits of targeting levels vs growth rates. They look at three types of targeting rules - growth-rate targeting, level targeting, and hybrid targeting - which says to keep the growth of nominal income as close to constant plus whatever percentage gap there is between real income and its equilibrium level.

While considering how to best implement the policies they make an analogy that nominal income's response to a central bank's purchases of securities is closer to that of steering a ship than that of steering an automobile. The Fed's ability to influence forecasts can help make nominal income growth more responsive to

current policy. Operating through consensus forecasts also allows nominal GDP targeting to be an accountable rule. While the Federal Reserve has never adopted explicitly nominal GDP targeting as a policy the authors find evidence in the data that the Federal Reserve may have adopted a hybrid type nominal GDP growth targeting policy in the 1970's and then growth rate targeting in the 1980's.

Finally Hall and Mankiw run simulations of a Phillips Curve with shocks meant to fit macroeconomic history from 1972-1991 where they allow for the three proposed nominal income targeting rules. They find that the volatility of the price level itself would have been much lower under any of the nominal income targeting rules. They find that level targeting performs the best in reducing volatility of the price level. The results are mixed and depend upon which target is chosen for the effect on inflation and output growth.

In McCallum and Nelson (1999a) the authors examine the merits of monetary policy targets of nominal GDP rather than the monetary aggregate or an inflation index. They first provide evidence that the Federal Reserve is actually engaged in nominal GDP targeting already by comparing the results of a regression of the federal funds rate on expected inflation and other variables to the results of regressing the federal funds rate on different forecasts of nominal GDP growth. They find that the goodness of fit improves as well as the precision of the coefficients on other variables in the regression when inflation is replaced with a measure of nominal GDP growth.

The authors examine the desirability of nominal income targeting in an open economy model with habit persistence and costly adjustment of prices and output by firms. The nominal income targeting rule that the authors impose on this system is a hybrid type rule in that it targets the growth of nominal GDP as well as the real output gap and it also has a lag of the interest rate in it to capture

interest rate smoothing. They find that the interest rate rule that targets nominal GDP leads to less volatility of inflation, output gap, nominal GDP growth, and interest rates than a traditional Taylor rule that responds to inflation.

Jensen (2002) compares the desirability of nominal GDP growth targeting with pure discretion and inflation targeting using the socially optimal policy as a benchmark. The policies considered are derived by minimizing loss functions with varying weights on inflation, the output gap, and nominal GDP growth. The desirability is weighted based upon performance in a social welfare function of the different policies in a standard linearized New Keynesian model.

Jensen finds that under the optimal targeting regimes when the economy is hit with technology shocks inflation targeting performs better - this is because under nominal income growth targeting a complete stabilization of the output gap leads to higher nominal income growth, and thus it leads to an inefficiency in responding to technology shocks from society's point of view. On the other hand when society is hit with cost-push shocks inflation targeting leads to inefficient outcomes while nominal income targeting is preferable.

More recently Billi (2013) examined the performance of nominal GDP level targeting in a New Keynesian model with the zero lower bound (ZLB). Nominal GDP level targeting is achieved by the central bank setting interest rates such that the squared deviations of the nominal GDP gap (the log price level plus the log output gap) from the targeted value for the nominal GDP gap is minimized. The author found that this policy performed better at exiting a ZLB episode than inflation targeting and also found that nominal GDP targeting is preferable in terms of welfare relative to inflation targeting. They were able to get a simple Taylor type rule with smoothing to produce similar results as the nominal GDP targeting rule.

Garín et al. (2015) looks at how well nominal GDP level targeting performs in a New Keynesian set-up that does not have the "divine coincidence". The authors use both a basic New Keynesian model with wage and price stickiness, and a medium-scale New Keynesian model that also includes capital and capital utilization costs, habit persistence, and investment adjustment costs. The authors assume that the central bank sets the interest rate to keep nominal GDP at target at all times. The authors report welfare deviations from the flexible price equilibrium and find that nominal GDP targeting consistently performs almost as well as output gap targeting, and performs better than inflation targeting or a Taylor rule. They also look at the case where the output gap is observed with noise and examine the degree of noise necessary for nominal GDP targeting to be just as good as output gap targeting in terms of welfare losses and find that it is fairly small - less than .66 percent standard deviation for any degree of persistence in the error that the output gap is observed.

Of interest to this paper is that nominal GDP level targeting is always determinate. This result can be compared with our results from Section 4.1 that the interest rate rule is determinate when the central bank responds more than one for one to deviations in nominal GDP from target. As Garín et al. (2015) note - a targeting regime that perfectly hits its target is equivalent to setting the coefficient on that targeted variable in an interest rate rule equal to infinity, which would be a special case of our results.

In Koenig (2012) the author examines the relationship between a monetary policy always adjusts to hit a nominal GDP level target and a Taylor rule in terms of achieving long-run stability of output and price expectations and argues that both fit the requirements for "flexible inflation targeting", ie holding inflation targets steady for the long term but being afforded discretion to respond to short-

run deviations in output. The key difference between the two policies is that under nominal GDP targeting the central bank will consider a longer history of price levels rather than just the current period's.

In a somewhat different context Sheedy (2014) examines nominal GDP targeting's effect on financial markets. In an overlapping generations model with debt and incomplete markets the author explores the welfare implications of different monetary policy rules. The author finds that a policy that perfectly hits its nominal GDP target can be used to "complete the market" by removing uncertainty about future nominal income that fall on the borrower. It is also shown that in the author's model inflation targeting is associated with greater volatility in the financial markets than nominal GDP targeting. Inflation targeting is actually pro-cyclical - credit is too freely available during expansions and it is too tight in recessions.

In McCallum (2015) the author looks at optimal monetary policy in a standard small-scale New Keynesian model. Nominal GDP targeting is achieved by minimizing a loss function. The author shows that one can rearrange the first order condition associated with a nominal GDP growth targeting policy to be nearly equivalent to the timeless perspective optimal monetary policy, policy that behaves the same way as the optimal policy under commitment but assuming that commitment had been adopted in the distant past.

Ball (1999) demonstrates that a nominal GDP targeting rule that targets either the level or that growth rate of nominal GDP perfectly will lead to non-stationarity in a purely backward-looking New Keynesian model. Ball looks at optimal monetary policy rules - monetary policy rules that minimize the variance of output and inflation. They define an efficient rule as a rule that achieves the minimized variance of output and inflation for some set of weights. In a closed

economy model with backward-looking IS and Phillips curve relations, the author demonstrates that nominal GDP targeting is an inefficient and even unstable policy in that it leads to an unbounded variance of both output and inflation due to the system displaying unit roots in its eigenvalues.

Yet McCallum (1997) demonstrates that with forward-looking inflation expectations, which is something more in line with the standard New Keynesian literature, the instability that Ball found is overturned. He demonstrates that Ball's result is unique to Ball's specification using a backward-looking Phillips Curve, independent of the behavior of aggregate demand, and not a problem for nominal GDP targeting as a policy.

### Learning and Monetary Policy

The first example where nominal GDP targeting is examined in the context of learning is in Mitra (2003). In a standard New Keynesian model with a central bank that pursues an interest rate policy that will reach its nominal GDP growth target every period he examines the determinacy of the steady state and the E-stability of the rational expectations equilibrium. This interest rate is found by setting nominal GDP growth equal to its target and then plugging in the IS and NKPC relations and solving for the nominal interest rate. He finds that the model's steady-state is determinate and the REE exhibits E-Stability. Mitra also finds that this is no longer true when the central bank assumes the agents in the economy make forecasts using rational expectations. Finally Mitra finds that when the central bank targets the expected nominal GDP growth the model becomes E-Unstable.

More recently Honkapohja and Mitra (2014) have compared the stability and expectational dynamics of inflation targeting, price level targeting, and a

Wicksellian interest rate rule that targets the level of nominal GDP. They use a New Keynesian set-up and assume that agents learn the steady-state values of output, inflation, and the nominal interest rate. They find that there is indeterminacy under the nominal GDP targeting interest rate rule if the rule responds to deviations from its target by more than the inverse of the discount rate times one plus the steady-state inflation rate. They examine the expectational stability of the steady state and find that for both rules the zero lower bound equilibrium is expectationally unstable. They also compare the characteristics of the expectational dynamics when the economy is away from the steady state. They find that inflation targeting converges to the steady state faster than price level or nominal GDP level targeting, but they find that nominal GDP level targeting reduces volatility of inflation and interest rates better than the other interest rate rules. Nominal GDP targeting is also shown to be second best to inflation targeting in minimizing output volatility. They also look at the expectational dynamics where agents have guidance about the policy rule when forming expectations, they find that guidance does speed up convergence for price level and nominal GDP level targeting, and improves the robustness of convergence as well.

This paper is also related to the broader literature on learning and monetary policy. Evans et al. (2008) examines the efficacy of monetary and fiscal policy in a New Keynesian model when the economy is trapped in a deflationary spiral or liquidity trap due to pessimistic shocks to expectations. They find that aggressive monetary policy alone is not enough to ensure that the economy does not enter a deflationary spiral. A combination of aggressive monetary policy and fiscal policy does however eliminate the risk of deflationary spirals by keeping inflation above some minimum threshold.



Bullard and Mitra (2002) examines the stability under learning of various Taylor-type rules. They view stability under learning as one condition for desirability of monetary policy and a more realistic measure than the determinacy of rational expectations equilibria. The authors look at four variants of the rules, rules that respond to contemporaneous output and inflation data, rules that respond to lagged output and inflation data, rules that respond to forecasts of future inflation and output, and rules that respond to expectations of current inflation and output. Under both contemporaneous data and contemporaneous expectations the authors find that the conditions for determinacy and E-Stability are the same, and similar to the Taylor principle. Under lagged data there can be situations where the system is determinate but E-Unstable. Under the rule that reacts to expectations of future output and inflation there can be regions that are E-Stable but not determinate.

Preston (2005) looks at the stability under learning of various interest rate policies when agents make decisions based on infinite horizon forecasts. The author argues that long horizon forecasts are more appropriate because agents are not able to make decisions based on the knowledge of the behavior of other agents in the economy like they would under rational expectations. As a result the model generates different predictions of the behavior of aggregate variables than under rational expectations. The interest rate rules considered are a nominal interest rate rule that depends on past exogenous disturbances, and a Taylor rule that responds to the model's endogenous variables. The author demonstrates that the former of the two rules is E-Unstable while the latter is E-Stable so long as it satisfies the Taylor principle.

Evans and Honkapohja (2003a) provides a survey of the general stability properties of various types of monetary policies under various information

assumptions. Evans and Honkapohja (2003b) look at the E-Stability of the optimal monetary policy rule under rational expectations and find that it is E-Unstable, but incorporating private agent's forecasts into the optimal policy rule overturns this.

This paper builds on these lines of research by investigating the determinacy of the steady state and E-stability of the REE in a New Keynesian model with nominal GDP growth rate feedback rules, which does not attribute a deep knowledge of the economy to the central bank.

This paper contributes both to the literature on nominal GDP growth and learning and monetary policy. We present both numerical results and parameter restrictions for the interest rate feedback rules that target nominal GDP growth that will ensure determinacy of the steady state and E-stability of the REE in a New Keynesian model. These rules react to deviations in either contemporaneous or expected nominal GDP growth from the growth of the flexible price level of real GDP.

### 2.3 Model

Consider the linearized New Keynesian model used in Clarida et al. (1999), which is derived from the behavior of utility-maximizing households and profit-maximizing firms in the Appendix:

$$x_t = -\phi(\hat{i}_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t \quad (2.1)$$

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t. \quad (2.2)$$

Here  $x_t$  is the output gap,  $\pi_t$  is inflation,  $\hat{i}_t$  is the deviation from steady state of the nominal interest rate and  $E_t^* x_{t+1}$  and  $E_t^* \pi_{t+1}$  represent the private-sector

expectations of the output gap and inflation in period  $t + 1$ . These expectations need not be rational expectations (RE). The model parameter  $\phi \geq 0$  is the intertemporal elasticity of substitution of consumption, which is equal to  $1/\sigma$  where  $\sigma$  is the representative household's CRRA utility parameter.  $\beta \in (0, 1)$  is the representative household's discount factor. Finally,  $\lambda \geq 0$  is the slope of the New Keynesian Phillips curve, which is a function of the steady-state level of markup on goods by the firms, the degree of price stickiness, the output elasticity of labor of the common production function, the household's CRRA utility parameter, and the household's disutility of labor parameter.

In most of the literature nominal GDP targeting is achieved by setting interest rates or money supply equal to the value that achieves the level or growth-rate target. This, however, requires the central bank to have knowledge of structural parameters to set the interest rate. We model the central bank as wanting to target nominal GDP growth by simply choosing to raise interest rates when nominal GDP growth is above the target growth rate and lowering interest rates when it is below the target growth rate. This is in contrast to Honkapohja and Mitra (2014) which has a rule that adjusts rates based upon levels of nominal GDP, Sheedy (2014) features an interest rate rule that responds to deviations in nominal GDP from target as well as changes in debt levels and the growth rate of real GDP, and McCallum and Nelson (1999a) which has the interest rate respond to nominal GDP growth with no target and the output gap separately, and a smoothing term.

A rule targeting nominal GDP growth could be thought of as a feedback rule similar in form to a Taylor rule. The central bank can choose how much it responds to deviations from the target. The central bank could choose to target a constant growth rate, but this does not take into account structural changes in

the economy which may cause potential GDP growth to be either greater or less than a constant target, so a central bank targeting nominal GDP growth would likely choose a “moving target” where the target changes based upon changes in what we call “potential” nominal GDP growth which is the growth in the “natural level” of real output plus trend inflation which is set to zero for this paper. We will consider rules that target both current and expected growth. We will also evaluate the E-Stability of the REE of the model with each of those rules under different information assumptions.

The rest of this section presents the form of these rules. The following section will present the determinacy and E-stability results associated with these rules.

### Nominal GDP Growth Rules

The NGDP rules under consideration can be boiled down to:

$$\hat{i}_t = \alpha E_t^*(\pi_{t+i} + x_{t+i} - x_{t+i-1}), \quad (2.3)$$

where  $i$  can take on the values  $\{0, 1, 2\}$  representing rules that resound to current and future NGDP growth. This rule says the central bank will raise interest rates when the growth rate of nominal GDP,  $\pi_{t+i} + y_{t+i} - y_{t+i-1}$ , is above the growth rate of the natural level of real output plus its target for inflation. Because we assume zero trend inflation we also assume that the central bank targets an inflation rate of zero.

For E-stability we will consider both case where the information set at time  $t$  contains all current variables and the case where the information set at time  $t$  contains time  $t$  exogenous variables ( $g_t$  and  $u_t$ ) and  $t - 1$  endogenous variables ( $x_{t-1}$  and  $\pi_{t-1}$ ).

## Nominal GDP Level Rules

We also consider the case where the central bank is targeting the level of NGDP:

$$i_t = \alpha E_t^*(p_{t+i} + x_{t+i}), \quad (2.4)$$

where  $p_t$  is the log price level and as with (2.3)  $i$  takes on the values of  $\{0, 1, 2\}$ . This rule tells the central bank to raise interest rates when the level of nominal GDP,  $p_{t+i} + y_{t+i}$ , rises above the natural level of real output where we have assumed that the central bank targets of price level of one.

We will analyze both information set assumptions as with the growth based rule from before.

As mentioned earlier, the majority of the rest of the literature with similar rules does not adjust rates based on the deviations from the natural level or has a generic target. The closest analog for our target is Mitra (2003). He implements nominal GDP growth targeting assuming that the central bank knows the structural relationships in the economy and assuming that interest rates are set by assuming that nominal GDP growth is equal to target. Mitra's targeting policy sets the growth of nominal GDP equal to the growth in real potential output.

## 2.4 Results

For macroeconomic models with rational expectations, there is a concern about the existence of multiple equilibrium paths, known as indeterminacy. Depending on the parameters of the model it may be the case that at any point in time any value of a free variable ( $x_t$  and  $\pi_t$  here) can be shown to be part of an equilibrium path to the steady state. In order to establish the uniqueness of the

Rational Expectations Equilibrium (REE) of this system we put each system in the form:

$$y_t = AE_t y_{t+1} + Bv_t. \quad (2.5)$$

For determinacy the eigenvalues of  $A$  that are inside the unit circle must be equal to the number of free variables and the number of eigenvalues of  $A$  that are outside the unit circle must be equal to the number of predetermined variables.

For E-stability rather than rational expectations we assume that agents are boundedly rational, in particular, that the agents use data generated by the economy described in the model to use recursive least squares to form their forecasts of future variables. We assume that agents use forecast rules that are of the same form as the minimum state variable (MSV) solution. We will describe the conditions for E-stability of the REEs in each of the subsections as they vary across the timing of the rules and the information assumptions used.

#### Current Nominal GDP Growth Rule

When the rule described in (2.3) with  $i = 0$  is used  $y_t = (x_t, \pi_t, x_{t-1})'$  and

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \beta & \lambda \\ -\frac{1}{\alpha\phi} & -\frac{1}{\alpha} + \beta & \frac{1+\alpha\phi+\alpha\lambda\phi}{\alpha\phi} \end{bmatrix}. \quad (2.6)$$

For determinacy to hold the number of eigenvalues of  $A$  inside the unit circle must be equal to the number of free variables. Because there is one pre-determined variable,  $x_{t-1}$  and two free variables,  $x_t$  and  $\pi_t$ , for uniqueness of the REE two of the eigenvalues of  $A$  need to be inside the unit circle and one eigenvalue needs to

be outside of the unit circle. The analytic form of the eigenvalues of this matrix are quite complex, and analytic conditions for determinacy do not appear possible to obtain.

Notice that the rule is a special case of the optimal rule derived in McCallum and Nelson (2000) although only when  $\alpha > 1$  for the above rule and for specific values of  $\theta$ ,  $\omega$ , and  $\kappa$  and the AR coefficient on the lagged interest rate set to zero:

$$\hat{i}_t = \pi_t + \theta(\pi_t + (\omega/\kappa)(x_t - x_{t-1})) \quad (2.7)$$

where  $\omega$  is the relative weight placed on output variance in the central bank's objective function,  $\kappa$  is the slope of the Phillips curve when the NKPC is written in terms of  $y_t$  rather than  $x_t$ , and  $\theta$  in a similar role as  $\alpha$  in our model.

While McCallum and Nelson (2000) did not report results on determinacy of this rule, Evans and Honkapohja (2006) finds that numerically all parameter values of  $\kappa$  and  $\theta$  appeared to be determinate under the calibrations used. The results we obtain with (2.3) with  $i = 0$  when using the same calibrations: Clarida et al. (2000) -  $\beta = .99$ ,  $\phi = 1$ , and  $\lambda = .3$ ; Woodford (1999) -  $\beta = .99$ ,  $\phi = 6.11$ , and  $\lambda = .024$ ; McCallum and Nelson (1999b) -  $\beta = .99$ ,  $\phi = .164$ , and  $\lambda = .3$  are consistent with the findings of Evans and Honkapohja (2006).

We conjecture that the REE is not unique when  $\alpha < 1$  and is unique when  $\alpha > 1$ . This holds true in all three model calibrations, which is in line with the findings related to the optimal rule from McCallum and Nelson (2000) in Evans and Honkapohja (2006).

For E-stability, following Honkapohja and Evans (2003) the reduced form of our model can be written as:

$$y_t = ME_t^* y_{t+1} + Ny_{t-1} + Pv_t. \quad (2.8)$$

We assume that agents use forecast rules that are of the same form as the minimum state variable (MSV) solution which takes the following form:

$$y_t = a + by_{t-1} + cv_t. \quad (2.9)$$

We also assume that the agents forecast future variables based upon a perceived law of motion (PLM) in the same form as the MSV solution. Additionally we assume that their information set is  $(1, y_t, v_t)$ , therefore they form expectations:

$$E_t^* y_{t+1} = a_t + b_t y_t + c_t F v_t \quad (2.10)$$

where  $F$  is a diagonal matrix with the AR parameters for the two shocks along the diagonal which are set to .9 each following Mitra (2003). We assume that  $F$  is part of the agents information set.

Agents update their estimates of  $(a, b, c)$  every period using the following algorithm:

$$\begin{aligned} \zeta_t &= \zeta_{t-1} + t^{-1} R_t^{-1} e_{t-1} \zeta_t' \\ R_t &= R_{t-1} + t^{-1} (e_{t-1} e_{t-1}' - R_{t-1}) \end{aligned}$$

where  $\zeta_t' = (a_t, b_t, c_t)$ ,  $e_t' = (1, y_{t-1}', v_t')$ , and  $\varsigma_t = y_{t-1} - \zeta_{t-1}' e_{t-1}$ . We are now concerned with whether the above stochastic recursive algorithm will converge to the REE. Evans and Honkapohja (2001) shows that the above algorithm will



converge to the REE values if the REE is expectationally stable or E-stable, this is called the E-stability principal. What this means is that with a rule meant to revise parameters any perturbations away from the REE will return to the REE. To obtain the E-stability conditions first we'll plug equation (2.10) into (2.8) in order to obtain the actual law of motion (ALM):

$$y_t = (I - Mb)^{-1}Ma + (I - Mb)^{-1}Ny_{t-1} + (I - Mb)^{-1}(McF + P)v_t. \quad (2.11)$$

We can see in the above equation that there is a mapping of the PLM into the ALM, this map is:

$$T(a, b, c) = ((I - Mb)^{-1}Ma, (I - Mb)^{-1}N, (I - Mb)^{-1}(McF + P)). \quad (2.12)$$

The E-Stability of this system is governed by the following differential equation:

$$\frac{d}{d\tau}(a, b, c) = T(a, b, c) - (a, b, c). \quad (2.13)$$

Following Evans and Honkapohja (2001) in models of the form expressed above E-stability depends upon the eigenvalues of the following:

$$DT_a(\bar{a}, \bar{b}) = (I - M\bar{b})^{-1}M \quad (2.14)$$

$$DT_b(\bar{b}) = [(I - M\bar{b})^{-1}N] \otimes [(I - M\bar{b})^{-1}M] \quad (2.15)$$

$$DT_c(\bar{b}, \bar{c}) = F' \otimes [(I - M\bar{\beta})^{-1}M]. \quad (2.16)$$

With information set  $(y'_t, v'_t, 1)$  a necessary condition for E-stability is that the eigenvalues of  $DT_a$ ,  $DT_b$ , and  $DT_c$  have real parts less than 1.

To assess E-Stability we will have to find  $\bar{b}$  from the MSV solution. It can be shown that  $\bar{b}$  solves the matrix quadratic:

$$Mb^2 - b + N = 0 \quad (2.17)$$

We use the methods from McCallum (2011) to find the MSV solution for models of this form. The above matrix quadratic can be written as:

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} b^2 \\ b \end{bmatrix} = \begin{bmatrix} I & -N \\ I & 0 \end{bmatrix} \begin{bmatrix} b \\ I \end{bmatrix}. \quad (2.18)$$

Taking the generalized Schur decomposition of the first matrix on each side of the equation referred to as  $A$  and  $B$  respectively, we have that there is a  $Q$ ,  $Z$ ,  $S$ , and  $T$  such that  $QA = SZ$  and  $QB = TZ$ . Pre-multiplying both sides of the above equation by  $Q$  it will then be:

$$\begin{bmatrix} S_{1,1} & 0 \\ S_{2,1} & S_{2,2} \end{bmatrix} \begin{bmatrix} Z_{1,1} & Z_{1,2} \\ Z_{2,1} & Z_{2,2} \end{bmatrix} \begin{bmatrix} b^2 \\ b \end{bmatrix} = \begin{bmatrix} T_{1,1} & 0 \\ T_{2,1} & T_{2,2} \end{bmatrix} \begin{bmatrix} Z_{1,1} & Z_{1,2} \\ Z_{2,1} & Z_{2,2} \end{bmatrix} \begin{bmatrix} b \\ I \end{bmatrix}. \quad (2.19)$$

The first row is  $S_{11}(Z_{11}b + Z_{12})b = T_{11}(Z_{11}b + Z_{12})$ , giving  $b = -Z_{11}^{-1}Z_{12}$ . The diagonal elements of  $S$  and  $T$  together ( $s_{11}/t_{11}$ ) form the generalized eigenvalues of the matrix pencil  $(A - \lambda B)$ . When solving for  $b$ , we need to make sure that the generalized eigenvalues are ordered such that  $b \rightarrow 0$  as  $N \rightarrow 0$ .

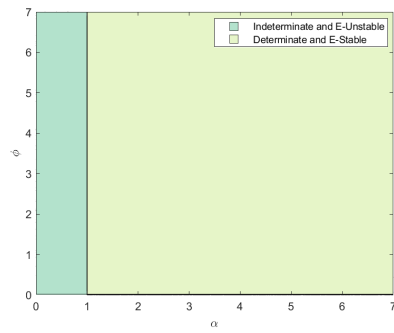
As with determinacy, we could not obtain analytic results for E-Stability. The graphs of the regions of determinacy and E-Stability in Figure 1 possess interesting

features. All figures fix  $\beta = .99$ , and in figure (1a) we fix  $\lambda = .3$ , meaning that at  $\phi = 1$  we get the results for the (Clarida et al., 2000) and at  $\phi = .164$  we get the results for McCallum and Nelson (1999b). We find that with a fixed  $\lambda$  and a free  $\phi$  whenever  $\alpha > 1$  the steady state is determinate and most of the determinate region is E-Stable although there is a sliver that is E-Unstable and determinate near  $\alpha$  equal to one. In figure (1b) when we instead fix  $\phi = .164$  and examine determinacy over  $\alpha$  and  $\lambda$ , we find again that determinacy and E-stability occur when  $\alpha > 1$  and also note that because  $\phi$  is sufficiently low there is no region that is E-Stable and indeterminate. Note that  $\lambda = .3$  in figure (1b) corresponds to the results for McCallum and Nelson (1999b). Finally in (1c) we fix  $\phi = 6.11$  as in Woodford (1999) to make sure that the results do not flip based upon whether  $\phi$  is greater or less than 1, notice that the results here are what we would expect based upon those in figure (1a), the steady state is determinate and E-stable, a wedge of E-instability and determinacy near  $\alpha = 1$ , and a section of E-Stability and indeterminacy when  $\alpha < 1$ .

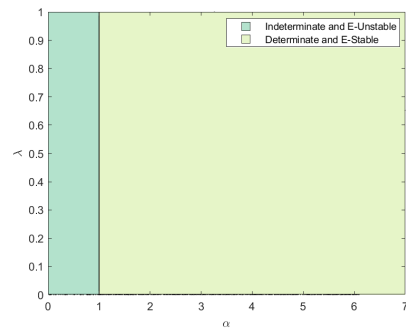
For context of these determinacy and E-Stability results, Mitra (2003) examines a rule that targets current period nominal GDP perfectly every period. He finds that the rule is always determinate and E-stable. For the rule considered here, determinacy depends upon the intensity of response to deviations from target and E-stability varies with the parameters and the intensity of response. Our results are similar to those found in Evans and Honkapohja (2006) for an interest rate rule based on the timeless perspective optimal monetary policy.

It is common to assume that the information set at time  $t$  is  $(1, y_{t-1}, v_t)$ . This assumption does not change the determinacy property nor the MSV solution. When this is assumed the interest rate rule now looks like:

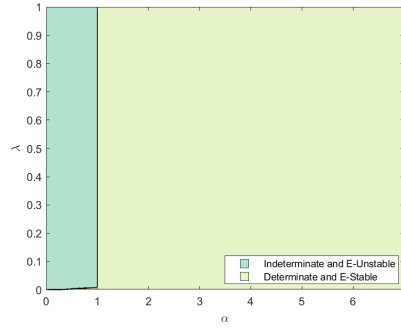
## Determinacy and E-Stability of Contemporaneous NGDP Growth Feedback Rule



(a)  $\beta = .99$  and  $\lambda = .3$



(b)  $\beta = .99$  and  $\phi = 0.164$



(c)  $\beta = .99$  and  $\phi = 6.11$

FIGURE 1. All three figures show the regions of determinacy and E-stability over different regions of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

$$\hat{i}_t = \alpha(E_t^* \pi_t + E_t^* x_t - x_{t-1}).$$

The system can be written as:

$$y_t = K E_t^* y_t + M E_t^* y_{t+1} + N y_{t-1} + P v_t. \quad (2.20)$$

Using a PLM based upon the form of the MSV solution the agents forecast future variables:

$$E_t^* y_{t+1} = a_t + b_t(a_t + b_t y_{t-1} + c_t v_t) + c_t F v_t \quad (2.21)$$

$$E_t^* y_t = a_t + b_t y_{t-1} + c_t v_t. \quad (2.22)$$

Agents update their estimates of  $(a, b, c)$  every period using the following algorithm:

$$\zeta_t = \zeta_{t-1} + t^{-1} R_t^{-1} e_{t-1} \zeta_t'$$

$$R_t = R_{t-1} + t^{-1} (e_{t-1} e_{t-1}' - R_{t-1})$$

Where  $\zeta_t' = (a_t, b_t, c_t)$ ,  $e_t' = (1, y_{t-1}', v_t')$ , and  $\varsigma_t = y_{t-1} - \zeta_{t-1}' e_{t-1}$ . As before the E-stability principal guarantees that this algorithm will converge to the REE over time. To obtain the E-stability conditions first we'll plug equations (2.21) and (2.22) into (2.20) in order to obtain the ALM:

$$y_t = (K + M(I + b))a + (N + Kb + Mb^2)y_{t-1} + (Kc + Mbc + McF + P)v_t. \quad (2.23)$$

We can see in the above equation that there is a mapping of the PLM into the ALM, this map is:

$$T(a, b, c) = ((K + M(I + b))a, N + Kb + Mb^2, Kc + Mbc + McF + P). \quad (2.24)$$

The E-Stability of this system is governed by the following differential equation:

$$\frac{d}{d\tau}(a, b, c) = T(a, b, c) - (a, b, c). \quad (2.25)$$

Following Evans and Honkapohja (2006) in models of the form expressed above E-stability depends upon the eigenvalues of the following:

$$DT_a(\bar{a}, \bar{b}) = K + M(I + b) \quad (2.26)$$

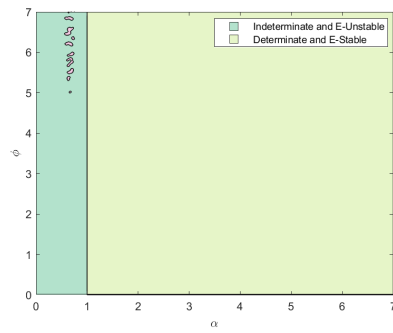
$$DT_b(\bar{b}) = \bar{b}' \otimes M + I \otimes M\bar{b} + I \otimes K \quad (2.27)$$

$$DT_c(\bar{b}, \bar{c}) = F' \otimes M + I \otimes M\bar{b} + I \otimes K. \quad (2.28)$$

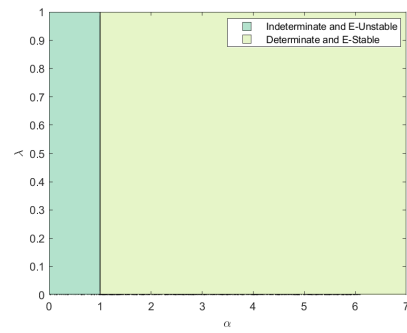
With information set  $(1, y'_{t-1}, v'_t)$  a necessary condition for E-stability is that the eigenvalues of  $DT_a$ ,  $DT_b$ , and  $DT_c$  have real parts less than 1.

Figure (2) is organized in a similar manner to figure (1). As is apparent there is not a large difference between the regions of determinacy and E-stability for either information set, however there is now a small set of values of the parameters that are E-stable and indeterminate in figure (2a) and (2c) implying stable sunspots.

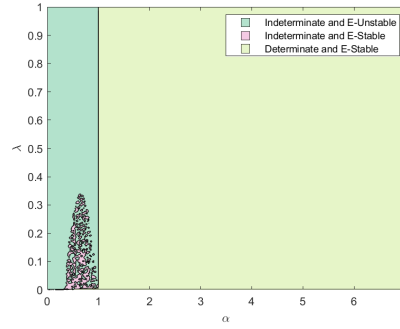
## Determinacy and E-Stability of Contemporaneous NGDP Growth Feedback Rule - Alternative Information



(a)  $\beta = .99$  and  $\lambda = .3$



(b)  $\beta = .99$  and  $\phi = 0.164$



(c)  $\beta = .99$  and  $\phi = 6.11$

FIGURE 2. All three figures show the regions of determinacy and E-stability over different regions of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

## Expected Nominal GDP Growth Rule

When the rule described in (2.3) is used with  $i = 1$  and  $y_t = (x_t, \pi_t)'$  then:

$$A = \begin{bmatrix} 1 & -\theta \\ \lambda & \lambda\theta + \beta \end{bmatrix} \quad (2.29)$$

$$\theta = \frac{\phi(1 - \alpha)}{1 - \phi\alpha}$$

If the eigenvalues of  $A$  are both inside the unit circle then the model is determinate. Some useful values of  $\alpha$  for determinacy and E-stability are:

$$\tilde{\alpha} = \frac{2 + 2\beta + \lambda\phi}{\phi(2 + 2\beta + \lambda)}$$

$$\alpha_+^* = \frac{1 + 2\sqrt{\beta} + \beta + \lambda\phi}{\phi(1 + 2\sqrt{\beta} + \beta + \lambda)}$$

$$\alpha_-^* = \frac{1 - 2\sqrt{\beta} + \beta + \lambda\phi}{\phi(1 - 2\sqrt{\beta} + \beta + \lambda)}$$

### Proposition 2.4.1

For  $\phi > 1$  both eigenvalues of  $A$  are inside the unit circle if  $\alpha \in (\tilde{\alpha}, 1)$ ,  
for  $\phi < 1$  both the eigenvalues of  $A$  are inside the unit circle if  $\alpha \in (1, \tilde{\alpha})$ .

**Proof** The eigenvalues of  $A$  are:

$$f(\alpha) = \frac{(1 + \beta)(\alpha\phi - 1) + \lambda\phi(\alpha - 1) + \sqrt{-4\beta(-1 + \alpha\phi)^2 + ((1 + \beta)(1 - \alpha\phi) + \lambda\phi(1 - \alpha))^2}}{2(\alpha\phi - 1)}$$

$$g(\alpha) = \frac{(1 + \beta)(\alpha\phi - 1) + \lambda\phi(\alpha - 1) - \sqrt{-4\beta(-1 + \alpha\phi)^2 + ((1 + \beta)(1 - \alpha\phi) + \lambda\phi(1 - \alpha))^2}}{2(\alpha\phi - 1)}$$



For  $\phi > 1$  it can be verified that  $f(1) = 1$  and  $g(1) = \beta$ . For  $\phi < 1$  it can be verified that  $f(1) = \beta$  and  $g(1) = 1$ . If  $\phi > 1$  then  $\tilde{\alpha} < 1$  and if  $\phi < 1$  then  $\tilde{\alpha} > 1$ . For  $\phi > 1$  we have  $f(\tilde{\alpha}) = -\beta$  and  $g(\tilde{\alpha}) = -1$ . If  $\phi < 1$  then  $f(\tilde{\alpha}) = -1$  and  $g(\tilde{\alpha}) = -\beta$ .

For  $\phi > 1$  we have  $\alpha_+^* < \alpha_-^* < 1$ , and for  $\phi < 1$  we have  $1 < \alpha_-^+ < \alpha_+^*$ .

The discriminant of the characteristic polynomial is zero when  $\alpha = \alpha_{\pm}^*$ , thus we can factor the discriminant into  $(\alpha - \alpha_-^*)(\alpha - \alpha_+^*)$ . For  $\phi > 1$ :  $f(\alpha_+^*) = g(\alpha_+^*) = -\sqrt{\beta}$  and  $f(\alpha_-^*) = g(\alpha_-^*) = \sqrt{\beta}$ . For  $\phi < 1$  the opposite. When:

$$\min \{\alpha_-^*, \alpha_+^*\} < \alpha < \max \{\alpha_-^*, \alpha_+^*\}$$

then the discriminant is negative and the values of  $f$  and  $g$  are complex. The modulus of all complex values of  $f$  and  $g$  equal  $\sqrt{\beta}$ . Therefore all values of  $\alpha$  that generate complex eigenvalues will be inside the unit circle.

Now observe that the derivatives of  $f$  and  $g$  are:

$$f'(\alpha) = \frac{\lambda\phi(\phi-1)((1+\beta)(\alpha\phi-1) + \lambda\phi(\alpha-1) + \sqrt{((1+\beta)(1-\alpha\phi) + \lambda\phi(1-\alpha))^2 - 4\beta(\alpha\phi-1)^2})}{2(\alpha\phi-1)^2\sqrt{((1+\beta)(1-\alpha\phi) + \lambda\phi(1-\alpha))^2 - 4\beta(\alpha\phi-1)^2}}$$

$$g'(\alpha) = \frac{\lambda\phi(\phi-1)((1+\beta)(1-\alpha\phi) + \lambda\phi(1-\alpha) + \sqrt{((1+\beta)(1-\alpha\phi) + \lambda\phi(1-\alpha))^2 - 4\beta(\alpha\phi-1)^2})}{2(\alpha\phi-1)^2\sqrt{((1+\beta)(1-\alpha\phi) + \lambda\phi(1-\alpha))^2 - 4\beta(\alpha\phi-1)^2}}$$

When  $\phi > 1$   $f'(\alpha)$  is greater than 0 when  $\alpha > \alpha_-^*$ . Because  $f(\alpha_-^*) = \sqrt{\beta}$ ,  $f(1) = 1$ , and because  $f$  is strictly increasing and continuous on the interval  $\alpha \in (\alpha_-^*, 1)$  we can conclude that the values of  $f$  are inside the unit circle for that interval. Similarly  $g'(\alpha) < 0$  on that same interval. Because  $g(\alpha_+^*) = \sqrt{\beta}$ ,  $g(1) = \beta$ , and  $g$  is continuous and strictly decreasing on the interval  $\alpha \in (\alpha_-^*, 1)$  we can conclude that the values of  $g$  are inside the unit circle on that interval.

When  $\phi > 1$ :  $f'(\alpha) < 0$  and  $g'(\alpha) > 0$  when  $\alpha < \alpha_+^*$  provided  $\alpha \neq \phi^{-1}$ . We have  $f(\alpha_+^*) = g(\alpha_+^*) = -\sqrt{\beta}$  and we also know that  $f(\tilde{\alpha}) = -\beta$  and  $g(\tilde{\alpha}) = -1$ . As before because  $f$  and  $g$  are continuous (it can be verified that  $\phi^{-1} < \tilde{\alpha}$  for  $\phi > 1$ )

and strictly decreasing and increasing respectively on the interval  $\alpha \in (\tilde{\alpha}, \alpha_+^*)$  the values of  $f$  and  $g$  must be less than one in absolute value.

For  $\phi < 1$  the argument is almost the same.  $f'(\alpha) > 0$  and  $g'(\alpha) < 0$  when  $\alpha < \alpha_-^*$ . Because  $f(1) = \beta$ ,  $g(1) = 1$ ,  $f(\alpha_-^*) = g(\alpha_-^*) = \sqrt{\beta}$ ,  $f$  and  $g$  are continuous and strictly increasing and decreasing respectively on the interval  $\alpha \in (1, \alpha_-^*)$  then the values of  $f$  and  $g$  must be less than one in absolute value.

When  $\alpha > \alpha_+^*$  and  $\phi < 1$ ,  $f'(\alpha) < 0$  and  $g'(\alpha) > 0$  provided  $\alpha \neq \phi^{-1}$ . Because  $f(\alpha_+^*) = g(\alpha_+^*) = -\sqrt{\beta}$ ,  $f(\tilde{\alpha}) = -1$ ,  $g(\tilde{\alpha}) = -\beta$ ,  $f$  and  $g$  are continuous (it can be verified that  $\tilde{\alpha} < \phi^{-1}$  when  $\phi < 1$ ) and strictly decreasing and increasing respectively over the interval  $\alpha \in (\alpha_+^*, \tilde{\alpha})$  then the values of  $f$  and  $g$  must be less than one in absolute value.

For  $\phi = 1$  the eigenvalues take on the following values:

$$\begin{aligned} & \frac{1 + \beta + \lambda}{2} + \frac{\sqrt{((1 + \beta + \lambda)^2 - 4\beta)}}{2} \\ & \frac{1 + \beta + \lambda}{2} - \frac{\sqrt{(1 + \beta + \lambda)^2 - 4\beta}}{2}. \end{aligned}$$

the larger of the two is never less than one given the parameter restrictions. A direct implication is that the calibration from Clarida et al. (2000) will always have an indeterminate equilibrium with the forward looking rule. To see why look at figure (3a). As  $\phi \rightarrow 1$  then  $\tilde{\alpha} \rightarrow 1$  this leads to there being no values of  $\alpha$  for which the eigenvalues of  $A$  are inside the unit circle.

Alternatively notice that when  $\phi = 1$  is used then (2.1) reduces to:

$$x_t = E_t x_{t+1} + E_t \pi_{t+1} + \frac{1}{1 - \alpha} g_t \quad (2.30)$$

Now  $\alpha$ 's only impact on the IS equation is on the effects of the exogenous process  $g_t$ , it does not have an effect how the output gap responds to expected inflation and the expected output gap. Finally we can also observe that  $\phi = 1$  also implies that  $\theta = 1$ .

Next we will look at the stability under least squares learning of the system when the interest rate targets expected nominal GDP growth. The system with this interest rate rule can be written as:

$$y_t = ME_t^* y_{t+1} + P v_t \quad (2.31)$$

The minimum state variable (MSV) solution takes the following form:

$$y_t = a + c v_t. \quad (2.32)$$

We assume that the agents forecast future variables based upon a perceived law of motion (PLM) in the same form as the MSV solution. Additionally we assume that their information set is  $(1, y_t, v_t)$ , therefore they form expectations:

$$E_t^* y_{t+1} = a_t + c_t F v_t. \quad (2.33)$$

Where  $F$  is a diagonal matrix with the AR parameters for the two shocks along the diagonal which are set to .9 each following Mitra (2003). We assume that  $F$  is part of the agents information set. Agents update their estimates of  $(a, c)$  every period using the following algorithm:

$$\begin{aligned}\zeta_t &= \zeta_{t-1} + t^{-1} R_t^{-1} e_{t-1} \zeta_t' \\ R_t &= R_{t-1} + t^{-1} (e_{t-1} e_{t-1}' - R_{t-1})\end{aligned}$$

Where  $\zeta_t' = (a_t, c_t)$ ,  $e_t' = (1, v_t')$ , and  $\varsigma_t = y_{t-1} - \zeta_{t-1}' e_{t-1}$ . We are now concerned with whether the above stochastic recursive algorithm will converge to the REE. To obtain the E-stability conditions first we'll plug equation (2.32) into (2.31) in order to obtain the actual law of motion (ALM):

$$y_t = Ma + (McF + P)v_t. \quad (2.34)$$

The mapping of the PLM into the ALM is:

$$T(a, c) = (Ma, McF + P). \quad (2.35)$$

The E-Stability of this system is governed by the following differential equation:

$$\frac{d}{d\tau}(a, c) = T(a, c) - (a, c). \quad (2.36)$$

Following Evans and Honkapohja (2001) in models of the form expressed above E-stability depends upon the eigenvalues of the following:

$$DT_a(\bar{a}) = M \quad (2.37)$$

$$DT_c(\bar{b}, \bar{c}) = F' \otimes M \quad (2.38)$$

The REE is E-Stable when the eigenvalues of  $DT_a$  and  $DT_c$  have real parts less than 1.

The unique eigenvalues of  $DT_c$  are the eigenvalues of  $DT_a$  scaled down by .9 and therefore we only need to concern ourselves with the eigenvalues of  $DT_a$  for E-stability. By Proposition 2.4.1 and noting that  $M$  is the matrix we referred to as  $A$  in the previous section, we already know that E-Stability holds for  $\alpha \in (\tilde{\alpha}, 1)$  for  $\phi > 1$  and  $\alpha \in (1, \tilde{\alpha})$  for  $\phi < 1$ . We also know that the value of  $\alpha$  will have no impact on E-Stability when  $\phi = 1$ .

#### Proposition 2.4.2

For  $\phi > 1$  the real parts of the eigenvalues of  $M$  are less than one for  $\alpha \in (\phi^{-1}, 1)$ , for  $\phi < 1$  the real parts of the eigenvalues of  $M$  are less than one for  $\alpha \in (1, \phi^{-1})$ .

**Proof** By Proposition 4.1 we have that both eigenvalues of  $M$  (hereafter referred to as  $f$  and  $g$  defined as they were in the proof to Propostion 4.1) are between negative one and one when  $\alpha \in (\tilde{\alpha}, 1)$  for  $\phi > 1$  and  $\alpha \in (1, \tilde{\alpha})$  for  $\phi < 1$ . Recall that if  $\phi > 1$  then  $g$  is strictly increasing when  $\alpha < \alpha_+^*$  except for  $\alpha = \phi^{-1}$ . The limit of  $g$  from the left and the right respectively are:

$$\lim_{\alpha \rightarrow \frac{1}{\phi}^+} \frac{(1 + \beta)(\alpha\phi - 1) + \lambda\phi - \sqrt{((1 + \beta)(1 - \alpha\phi) + \lambda\phi(1 - \alpha))^2 - 4\beta(\alpha\phi - 1)^2}}{2(\alpha\phi - 1)} = -\infty$$

$$\lim_{\alpha \rightarrow \frac{1}{\phi}^-} \frac{(1 + \beta)(\alpha\phi - 1) + \lambda\phi - \sqrt{((1 + \beta)(1 - \alpha\phi) + \lambda\phi(1 - \alpha))^2 - 4\beta(\alpha\phi - 1)^2}}{2(\alpha\phi - 1)} = \infty$$

The first limit combined with the fact that  $g$  is strictly increasing and continuous on the interval  $\alpha \in (\phi^{-1}, \tilde{\alpha})$ , equal to negative one when  $\alpha = \tilde{\alpha}$ , and the results of Proposition 4.1 imply that the real part of  $g$  is less than 1 for  $\alpha \in (\phi^{-1}, 1)$ .

Next observe that:

$$g(0) = \frac{1 + \beta + \lambda + \sqrt{(1 + \beta + \lambda)^2 - 4\beta}}{2}$$

Which is greater than one given our assumptions on  $\beta$  and  $\lambda$ . Because  $g$  is strictly increasing and continuous for  $\alpha \in [0, \phi^{-1}]$   $g$  is greater than one for all values of  $\alpha$  on that interval.

Finally  $f$  is strictly decreasing for all values of  $\alpha < \alpha_+^*$  except for  $\alpha = \phi^-$ , and the limit of  $f$  as  $\alpha \rightarrow \phi^{-1}$  is zero by L'Hospital's rule. As a result the real part of  $f$  is less than 1 for  $\alpha \in (\phi^{-1}, 1)$ .

For  $\phi < 1$   $f$  is strictly decreasing for  $\alpha > \alpha_+^*$  and the limits from the left and right as it approaches  $\phi^{-1}$  are:

$$\begin{aligned} \lim_{\alpha \rightarrow \frac{1}{\phi}^+} \frac{(1 + \beta)(\alpha\phi - 1) + \lambda\phi - \sqrt{((1 + \beta)(1 - \alpha\phi) + \lambda\phi(1 - \alpha))^2 - 4\beta(\alpha\phi - 1)^2}}{2(\alpha\phi - 1)} &= \infty \\ \lim_{\alpha \rightarrow \frac{1}{\phi}^-} \frac{(1 + \beta)(\alpha\phi - 1) + \lambda\phi - \sqrt{((1 + \beta)(1 - \alpha\phi) + \lambda\phi(1 - \alpha))^2 - 4\beta(\alpha\phi - 1)^2}}{2(\alpha\phi - 1)} &= -\infty \end{aligned}$$

The second limit along with  $f(\tilde{\alpha}) = -1$ , that  $f$  is strictly decreasing for  $\alpha \in [\tilde{\alpha}, \phi^{-1})$  and the results from Proposition 4.1 imply that the real part of  $f$  is less than one for  $\alpha \in (1, \phi^{-1})$ . Together the first limit, that  $f$  strictly is decreasing and continuous for  $\alpha \in (\phi^{-1}, \infty)$  and that the limit of  $f$  as  $\alpha \rightarrow \infty$  is:

$$\frac{1 + \beta + \lambda + \sqrt{(1 + \beta + \lambda)^2 - 4\beta}}{2}$$

Implies that the real part of  $f$  is strictly greater than one for  $\alpha \in (\phi^{-1}, \infty)$ .

Finally observe that  $g$  is strictly increasing and continuous for  $\alpha > \alpha_+^*$  except for  $\alpha = \phi^{-1}$ ,  $g(\tilde{\alpha}) = -\beta$ , and that by l'Hospital's rule the limit of  $g$  as  $\alpha \rightarrow \phi^{-1}$  is

zero. These along with the results from Proposition 4.1 imply that the real part of  $g$  is less than one for  $\alpha \in (1, \phi^{-1})$ .

The results from Propositions 4.1 and 4.2 can be seen in Figure (3). As with previous figures, (3a) fixes  $\lambda = .3$  and plots the regions of determinacy and E-Stability in  $(\alpha, \phi)$  space. The region of E-Stability is larger than the region of determinacy which is contained in the E-Stable region (notice the small slivers on the edges). Comparing with (3b) and (3c) this result is much more pronounced. Particularly because the boundaries for E-Stability only depend on  $\phi$  whereas the boundaries for determinacy depend on  $\beta$ ,  $\phi$ , and  $\lambda$ . This differs from Mitra (2003) results for a rule that targeted expected nominal GDP growth, where the rule was always E-unstable.

The flipping of regions of determinacy around  $\phi = 1$  is similar to the results in Bilbiie (2008). The author presents a New Keynesian model with limited asset market participation in the form of a fixed fraction of households optimizing a static problem every period and not participating in asset markets. He assumes the central bank sets interest rates based on a linear combination of expected inflation and the current output gap. This eventually affects the coefficient on the real interest rate in the NKIS equation. Specifically, there is a threshold value for the fraction of households who do not participate in asset markets which causes the sign on that coefficient changes. The author refers to situations where the real interest rate has a positive effect on real output as cases of “Inverted Aggregate Demand Logic” (IADL). Under IADL the Taylor principle becomes inverted and the equilibrium becomes unique when the central bank responds to inflation expectations in a passive manner. The coefficient on expected inflation in the IS equation in their model will equal  $-\delta^{-1}(\phi_\pi - 1)$ , where  $-\delta^{-1}$  is the coefficient on the real interest rate and  $\phi_\pi$  is the response parameter in the central bank’s interest

TABLE 1. Inverted Aggregate Demand Logic and Determinacy

$\phi$	$\alpha$	Result
$< 1$	$< 1$	SADL violates Taylor Principle
$< 1$	$(1, \phi^{-1})$	SADL satisfies Taylor Principle
$< 1$	$> \phi^{-1}$	IADL and violates Inverted Taylor Principle
$> 1$	$> 1$	IADL and violates Inverted Taylor Principle
$> 1$	$(\phi^{-1}, 1)$	IADL and satisfies the Inverted Taylor Principle
$> 1$	$< \phi^{-1}$	SADL violates Taylor Principle

rate rule. An active response ( $\phi_\pi > 1$ ) under IADL  $\delta < 0$  will result in a positive response for real output. This, in turn, leads to an increase in inflation today and the non-fundamental shock to inflation expectations becomes self-fulfilling. Instead, a passive response ( $\phi_\pi < 1$ ) is needed to ensure that inflation expectations do not become self-fulfilling.

To see how this relates to our results plus (2.3) for  $i = 1$  into the IS relation and solve for  $x_t$ . In this case, we get:

$$x_t = E_t^* x_{t+1} - \frac{\phi(\alpha - 1)}{1 - \phi\alpha} E_t^* \pi_{t+1} + g_t \quad (2.39)$$

We can look case by case for when IADL shows up and whether that case results in a determinate equilibrium or not. We see that the determinacy results found in this section are in line with the results found for models that exhibit IADL.

As before we want to investigate how the results compare when we assume that the information set only includes lagged endogenous variables. With this new assumption the forward looking interest rate rule becomes:

$$\hat{i}_t = \alpha(E_t^* \pi_{t+1} + E_t^* x_{t+1} - E_t^* x_t) \quad (2.40)$$

Substituting (2.40) into (2.1):



## Determinacy and E-Stability of Expected NGDP Growth Feedback Rule

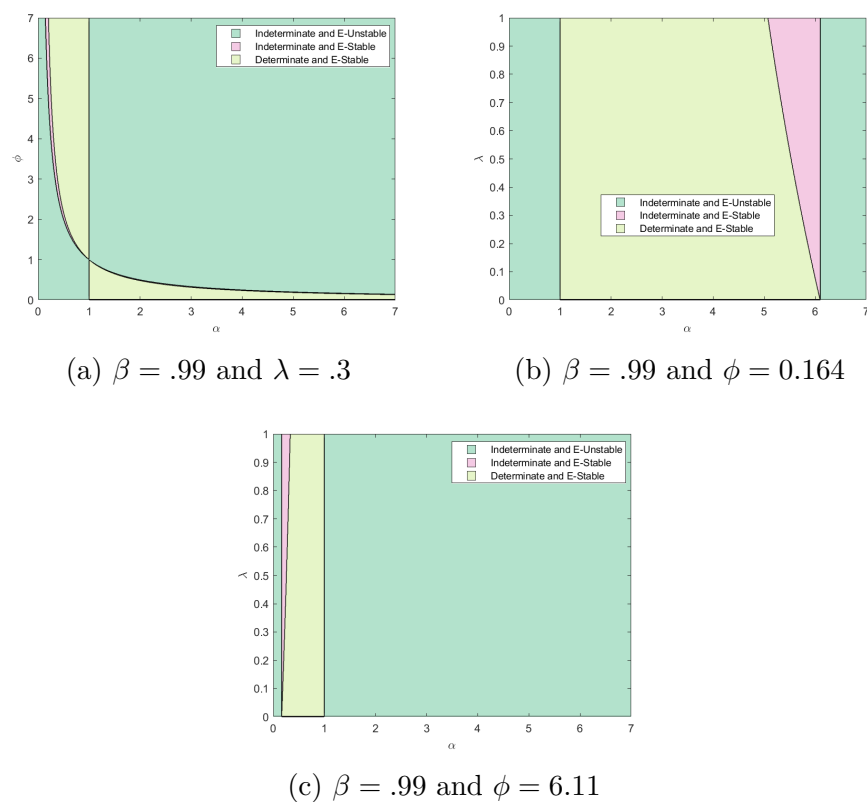


FIGURE 3. All three figures show the regions of determinacy and E-stability over sections of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

$$x_t = -\phi(\alpha - 1)E_t^* \pi_{t+1} + (1 - \phi\alpha)E_t^* x_{t+1} + \phi\alpha E_t^* x_t + g_t \quad (2.41)$$

This equation along with equation (2.2) can be rewritten in matrix form as:

$$y_t = KE_t^* y_t + ME_t^* y_{t+1} + Pv_t \quad (2.42)$$

The MSV solution and PLM are the same as in the previous section. The ALM however is now:

$$y_t = (K + M)a + (McF + Kc + P)v_t \quad (2.43)$$

This means the mapping from PLM to ALM is:

$$T(a, c) = (Ma + Ka, McF + Kc) \quad (2.44)$$

The stability is governed by the differential equation:

$$\frac{d}{d\tau}(a, c) = T(a, c) - (a, c) \quad (2.45)$$

E-stability depends upon the eigenvalues of the following:

$$DT_a(\bar{a}) = M + K \quad (2.46)$$

$$DT_c(\bar{a}, \bar{c}) = F' \otimes M + I_2 \otimes K \quad (2.47)$$

The inequalities describing when the model is E-Stable are not simple nor intuitive involving many different cases and sub-cases. Figure 4 illustrates the results in the same format as was used in the previous sections. (4a) shows

determinacy and E-Stability over  $\phi$  and  $\alpha$  when  $\lambda$  is fixed at .3. Note that as  $\phi$  increases the lowest value of  $\alpha$  that results in E-Stability also increases. Determinacy and E-stability differ so much only in a small region with  $\phi < 1$  and  $\alpha > 1$  is both determinate and E-Stable. In (4b) we fix  $\phi = .164$  and let  $\lambda$  float. Here we conjecture that for any  $\lambda$  the model is E-Stable for all values of  $\alpha > 1$ , and the determinacy results are similar to before. In (4c) we fix  $\phi$  at 6.11. Here we conjecture that for sufficiently low  $\lambda$  the model is E-Unstable for all values of  $\alpha$ . There also do not appear to be any values of  $\alpha$  that are both determinate and E-Stable which is in line with what we observe in (4a).

#### Two Period Ahead Expected Nominal GDP Growth Rule

To assess determinacy under the two period ahead NGDP growth rule we set  $y_t = (x_t, \pi_t, x_{t+1}, \pi_{t+1})$  and include two identity equations. We were unable to find analytic conditions for determinacy, however in figure 5 it appears that the REE is determinate for all parameter values considered.

There are however analytic conditions for E-stability. Now we write the system in the form:

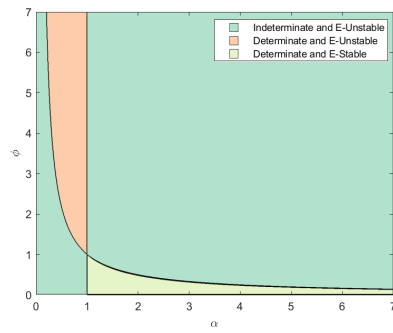
$$y_t = ME_t^* y_{t+1} + KE_t^* y_{t+2} + Pv_t. \quad (2.48)$$

Now the T-map becomes:

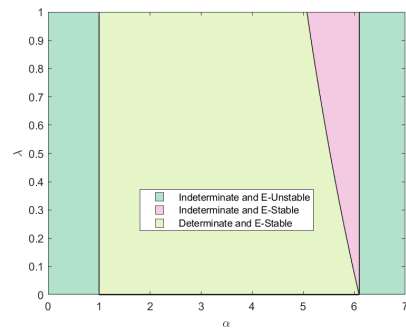
$$T(a, c) = ((M + K)a, McF + KcF^2). \quad (2.49)$$

E-stability depends upon the eigenvalues of the following having real parts less than one:

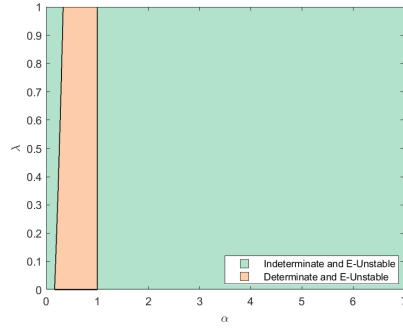
## E-Stability of Forward Looking NGDP Growth Feedback Rule - Alternative Information



(a)  $\beta = .99$  and  $\lambda = 0.3$



(b)  $\beta = .99$  and  $\phi = 0.164$



(c)  $\beta = .99$  and  $\phi = 6.11$

FIGURE 4. All three figures show the regions of determinacy E-Stability over different regions of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

$$DT_a = M + K, \quad (2.50)$$

$$DT_c = F' \otimes M + (F^2)' \otimes K. \quad (2.51)$$

While conditions for  $DT_a$  having eigenvalues with real parts less than one, the eigenvalues of  $DT_c$  do not appear to have conditions that are able to express or interpret. In general, it appears to be necessary that  $\alpha > 1$ , although for low values of  $\phi$  and high values of  $\lambda$  this appears to increase to  $\alpha > 2$ . On the other hand for larger values of  $\phi$  and low values of  $\lambda$  the REE is E-unstable for all values of  $\alpha$  considered. Using the alternative information set leads to all values of  $\alpha$  resulting in E-instability.

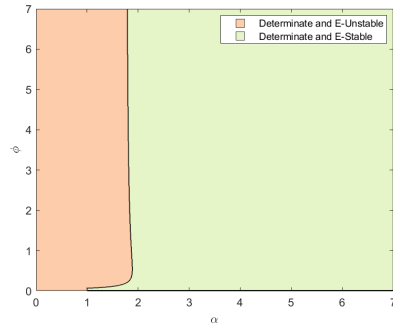
#### Current Nominal GDP Level Rule

When the rule (2.4) is used with  $i = 0$  we set  $y_t = (x_t, \pi_t, p_t)'$ . Now the system of equations includes an identity for the price level.

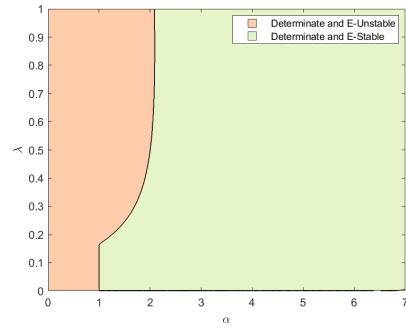
$$p_t = p_{t-1} + \pi_t$$

Following the results of Giannoni (2014), we know that determinacy holds for all values of  $\alpha > 0$ . While there is no obvious way to extend this analytical result to E-stability, it does appear that the same condition holds for E-stability as well. These results are robust to both information assumptions. This makes the rule desirable at least from the perspective that no matter how aggressive the central bank is about achieving its target, the REE will always be both determinate and E-stable.

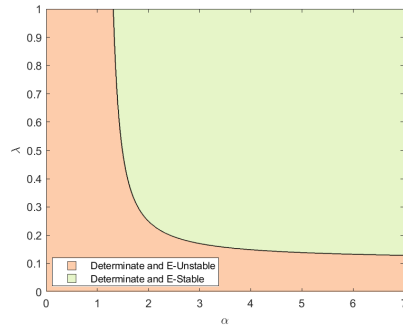
## E-Stability of Two Period Ahead Expected NGDP Feedback Rule



(a)  $\beta = .99$  and  $\lambda = 0.3$



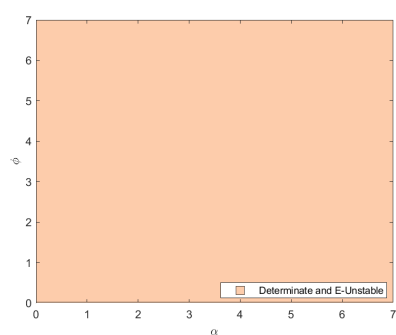
(b)  $\beta = .99$  and  $\phi = 0.164$



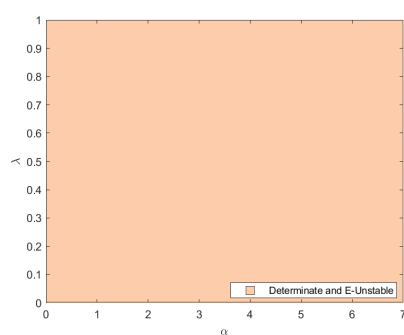
(c)  $\beta = .99$  and  $\phi = 6.11$

FIGURE 5. All three figures show the regions of determinacy E-Stability over different regions of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

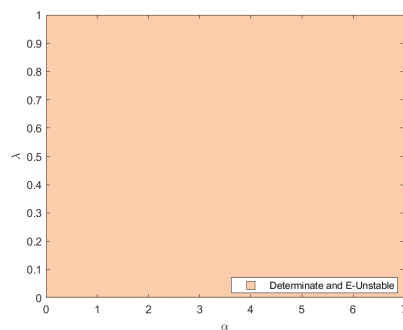
# **E-Stability of Two Period Ahead Expected NGDP Feedback Rule: Alternative Information**



(a)  $\beta = .99$  and  $\lambda = 0.3$



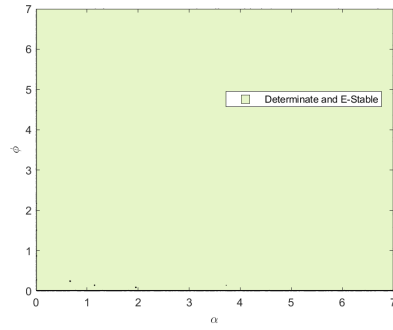
(b)  $\beta = .99$  and  $\phi = 0.164$



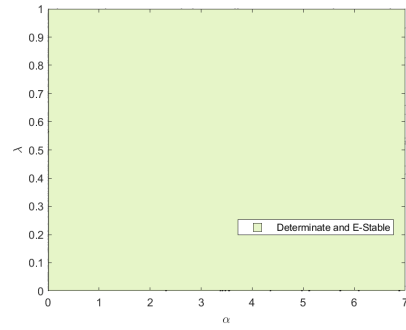
(c)  $\beta = .99$  and  $\phi = 6.11$

FIGURE 6. All three figures show the regions of determinacy E-Stability over different regions of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

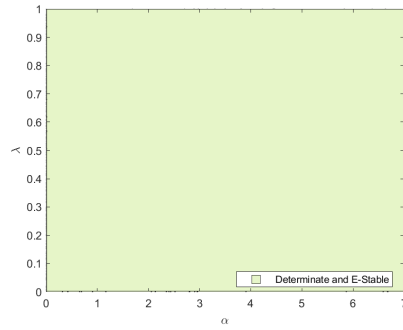
## E-Stability of NGDP Level Feedback Rule



(a)  $\beta = .99$  and  $\lambda = 0.3$



(b)  $\beta = .99$  and  $\phi = 0.164$

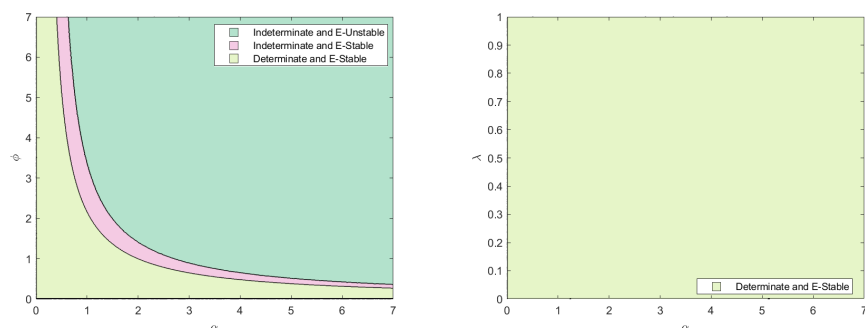


(c)  $\beta = .99$  and  $\phi = 6.11$

FIGURE 7. All three figures show the regions of determinacy E-Stability over different regions of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

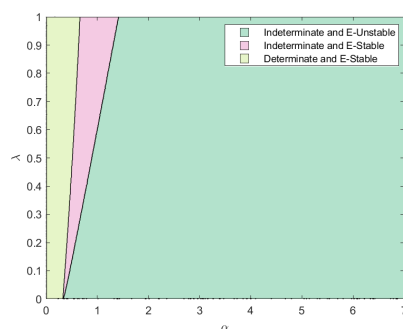


## E-Stability of Expected NGDP Level Feedback Rule



(a)  $\beta = .99$  and  $\lambda = 0.3$

(b)  $\beta = .99$  and  $\phi = 0.164$



(c)  $\beta = .99$  and  $\phi = 6.11$

FIGURE 8. All three figures show the regions of determinacy E-Stability over different regions of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

## One Period Ahead Nominal GDP Level Rule

The interest rate rule which targets nominal GDP's level one period ahead we can see that higher values for  $\phi$  lead to a very small range of values of  $\alpha$  that result in a determinate and E-stable equilibrium while for low values of  $\phi$  the range becomes quite large.

## Two Period Ahead Nominal GDP Level Rule

Under the interest rate rule that targets the level of nominal GDP two periods in the future, the system described in (2.5) will have  $y_t = (x_t, \pi_t, x_{t+1}, \pi_{t+1}, p_{t+1})$  with identity equations for  $x_{t+1}$  and  $\pi_{t+1}$ . The price level equation is written for  $p_{t+2}$ . We were unable to find analytic results for determinacy, but all parameterizations considered result in a determinate REE.

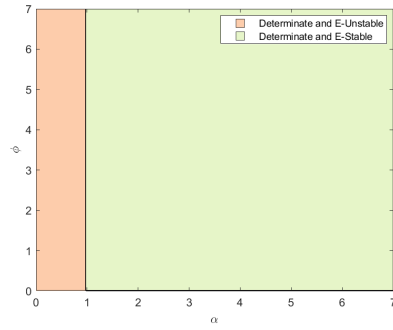
The system of equations used to study E-stability is in the same form as the one used for the two period ahead NGDP growth rule but now with  $y_t = (x_t, \pi_t, p_t)$ . The numerical results show that for most parameterizations  $\alpha > 1$  ensures E-stability of the REE, with some deviations for extreme values of  $\lambda$ .

Similar to the two period ahead NGDP growth rule, using alternate information leads to most but for this rule note all values of  $\alpha$  resulting in E-instability. We do find that for sufficiently small values of  $\phi$  and  $\lambda$  there are values of  $\alpha$  which result in E-stability.

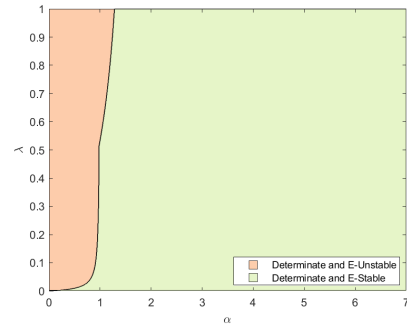
## 2.5 Conclusion

We have analyzed determinacy and E-Stability of equilibria when feedback rules that respond to nominal GDP growth and level are used. When the rule responds to contemporaneous nominal GDP growth the model is determinate and E-Stable only when the response to deviations of nominal GDP growth away from its target is at least greater than one for one. We also found that targeting current NGDP level leads to determinacy and E-stability for all parameterizations considered. When the interest rate responds to expectations of nominal GDP growth it depends on information assumptions. If the central bank is assumed to know the current period output gap then determinacy and E-Stability regions

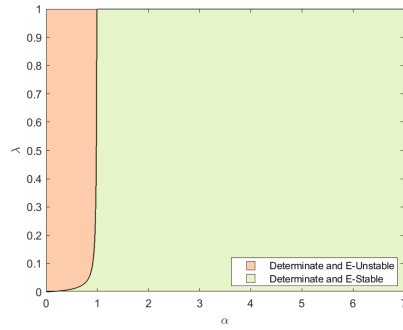
## E-Stability of Two Period Ahead NGDP Level Feedback Rule



(a)  $\beta = .99$  and  $\lambda = 0.3$



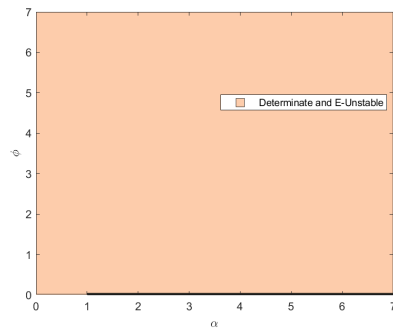
(b)  $\beta = .99$  and  $\phi = 0.164$



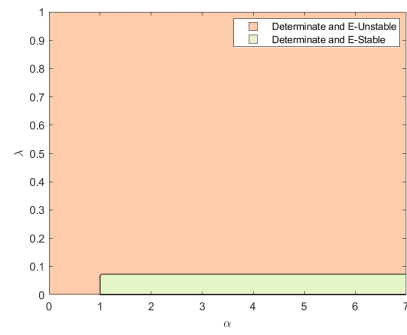
(c)  $\beta = .99$  and  $\phi = 6.11$

FIGURE 9. All three figures show the regions of determinacy E-Stability over different regions of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

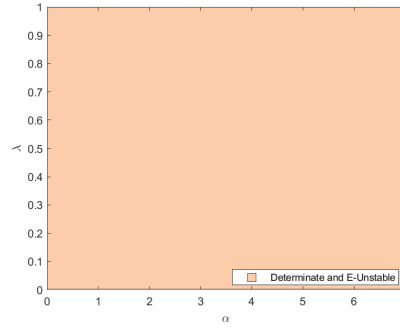
## E-Stability of Two Period Ahead NGDP Level Feedback Rule: Alternative Information



(a)  $\beta = .99$  and  $\lambda = 0.3$



(b)  $\beta = .99$  and  $\phi = 0.164$



(c)  $\beta = .99$  and  $\phi = 6.11$

FIGURE 10. All three figures show the regions of determinacy E-Stability over different regions of the parameter space. In all figures  $\beta = .99$ . In figure (a)  $\lambda$  is fixed at .3. In figure (b)  $\phi$  is fixed at 0.164. In figure (c)  $\phi$  is fixed at 6.11.

depend greatly on the size of the intertemporal elasticity of substitution of consumption parameter. This rule can also introduce something known as “inverted aggregate demand logic” which also inverts the Taylor principle. When it is assumed however that the central bank uses private agents forecasts of the current output gap then the results are difficult to completely characterize. It appears that when the proportions of the intertemporal elasticity of substitution and the slope of the Phillips Curve are correct the model is E-stable for all values of  $\alpha > 1$ , however, when  $\phi$  is too large relative to  $\lambda$  like in Woodford (1999) then the model may never be E-Stable with this interest rate rule. Ultimately a central bank choosing this rule should be careful to ensure they are responding the right amount to nominal GDP growth when setting interest rates. Additionally before jumping to serious policy prescriptions we would need to consider the effects of a lagged impact of policy changes.

## CHAPTER III

### NGDP TARGETING AND FINANCIAL FRICTIONS

#### 3.1 Introduction

Nominal GDP (NGDP) has been suggested yet not implemented as a nominal anchor for the central bank. The alleged benefits are the built-in countercyclical monetary policy, attractive zero lower bound properties, and its interaction with debt. We will be most concerned about the interaction between NGDP targeting and debt.

Under NGDP targeting, if NGDP falls either due to a fall in output or a fall in prices this causes the central bank to expand the money supply and lower interest rates increasing output and prices and bringing NGDP towards its target.

With regard to the zero lower bound a strict NGDP level target implies a very similar path for nominal interest rates as the optimal monetary policy. This is a policy involving rates being held at zero past the point that inflation and output start taking off.

Finally NGDP targets imply an interesting relationship with aggregate debt. Because NGDP targeting leads to higher inflation when real output is low it also lowers the real value of nominal debt contracts. When real output is lower that implies that households have less real resources available, so lowering the real value of debt helps debtors when times are tough. In addition to this desirable feature, Sheedy (2013) shows that in an overlapping generations model with incomplete markets an appropriate NGDP target "completes" the market by reducing uncertainty about future nominal income.

We assess whether NGDP targeting may have desirable traits when financial frictions are present. We examine the following six NGDP based monetary policies: i.) NGDP growth targeting interest rate rule, ii.) NGDP level targeting interest rate rule, iii.) optimized version of i, iv.) optimized version of ii, v.) NGDP growth targeting, and vi.) NGDP level targeting. In i-iv the central bank sets the interest rate in proportion to deviations of the targeted value from the target while in v-vi the central bank can perfectly achieve its targets.

The framework we use to examine the performance of these policies is the model developed in Cúrdia and Woodford (2016). In this model there are two types of households, patient and impatient. The patient households have a lower marginal utility of consumption than the impatient households, as a result the patient households save so that the impatient households may borrow to finance consumption. Due to the costs associated with creating loans and fraud borrowers pay a higher interest rate on loans than savers receive. Other than these modifications the model closely resembles the standard New Keynesian (NK) model.

This paper relates to two different strands of the literature. First this is intended to add to the understanding of the desirability of monetary policy that targets NGDP's level or growth rate. It also builds upon the literature focused on debt and financial frictions in DSGE models.

Federal Reserve Bank of San Francisco President John Williams (Williams, 2016) identifies NGDP targeting as a policy that would have beneficial properties in a world with a low natural rate of interest. He cites the fact that the policy delivers the “lower for longer” property of the optimal response at the zero lower bound, it guards against debt deflation, and a low natural rate of interest leads to more

inflation under NGDP targeting, which leaves more room for monetary policy to respond to negative shocks.

Sumner (2012) also has argued for the merits of NGDP targeting. In particular that the emphasis on inflation as a target made central banks' response to the Great Recession less expansionary than it should have been. Had central banks focused on stabilizing NGDP they would have responded to the recession much more aggressively. He also argues that it may stabilize asset markets. It could limit periods of extreme NGDP growth which are tied to bubble formation. It also helps limit the severity of drops in NGDP which can trigger financial crises. Sumner supports this with the fact that the first financial crisis around the Great Depression occurred one year into the depression.

Sumner also claims that an NGDP target is easy for the public to understand and Woodford (2012) agrees. Woodford also argues that the NGDP strikes the best balance between implementing optimal monetary policy and choosing a policy that can be practically explained, verified, and implemented.

We find that in our model most NGDP targeting policies would provide preferable outcomes to more traditional policies with a few exceptions. One of these exceptions is an interest rate rule that adjusts based upon NGDP growth. We use impulse response graphs and present the unconditional variance of the endogenous variables to help understand the reasons why some rules are preferred to others.

The rest of the paper is broken down as follows: We start with a literature review in section 3.2 in section 3.3 we present the linearized NK model with financial frictions derived in Cúrdia and Woodford (2016). Then in section 3.4 we present and explain the NGDP targeting policies as well as the more conventional policies. We present welfare calculations, impulse response graphs and



interpretations and unconditional variances for all the policies under consideration in section 3.5.

### 3.2 Literature Review

Sheedy (2014) looks at a topic similar to this paper. Sheedy demonstrates that in the presence of incomplete markets NGDP targeting makes households better off by making the nominal income more predictable. By allowing for prices to move counter-cyclically the policy stabilizes the debt to GDP ratio which ensures effective risk sharing. When real GDP falls inflation must increase which reduces the real value of debt in proportion to the fall in real GDP. This helps to better share risk amongst households.

In Garín et al. (2016) the authors compare the welfare losses of NGDP targeting with inflation targeting, output gap targeting and a Taylor rule in a NK model featuring price and wage rigidities. They find that NGDP targeting outperforms inflation targeting and the Taylor rule and is slightly worse than output gap targeting unless the output gap is observed with noise. They also compare the same three policies in an estimated medium-scale DSGE model featuring capital, habit formation for consumption, variable capital utilization, and indexation of wages and prices to past inflation. They find that the results from the simpler model still hold under the more realistic model.

Jensen (2002) uses a linearized NK model featuring output and inflation persistence to compare the welfare properties of policies derived from quadratic loss functions that either target inflation or NGDP growth targeting. The author finds that so long as there are shocks that pose a trade-off to stabilizing output and inflation, NGDP growth targeting results in lower welfare losses than inflation targeting.

Looking a bit further back into the literature Hall and Mankiw (1994) conduct simulations to examine the performance of different NGDP targeting rules at reducing the standard deviation of key macroeconomic variables. They consider level and growth targeting as well as a hybrid which targets growth and adjusts the target based on the output gap. When real data is used to generate policy errors in implementation they find that the price level would have significantly lower standard deviations under all three policies than it experienced in the real world. The level and hybrid targets lead to are able to achieve lower standard errors in inflation as well. And the hybrid target achieves a lower standard deviation in the output gap. All NGDP policies lead to a higher standard deviation of output growth. Under no policy error the policies lead to lower standard deviations of all of the relevant macroeconomic variables.

In Billi (2013) the author explores the ability of NGDP targets to both avoid zero lower bound episodes and mitigate their effects. The paper compares optimal discretionary NGDP level targets to optimal discretionary flexible inflation targeting and it compares a Taylor rule and an NGDP interest rate rule both subject to the zero lower bound. While inflation targeting requires positive trend inflation in order to avoid a deflationary spiral, even zero trend inflation under NGDP targeting avoids a deflationary spiral and NGDP targeting results in lower welfare loss relative to the Ramsey plan than inflation targeting. During a zero lower bound episode NGDP targeting raises inflation higher than an inflation targeting policy would, which leads to a faster recovery that would otherwise happen. Inflation targeting however does not allow inflation to rise above its target. For the interest rate rules the author finds that the difference in welfare loss between the rules are small when interest rate smoothing is present.

McCallum (2015) shows that NGDP growth targeting is a special case of the timeless optimal monetary policy with commitment. In McCallum and Nelson (1999a) the authors compare the standard deviations of inflation, the output gap, NGDP, and the nominal interest rate under inflation targeting and NGDP targeting in an open-economy NK model. The policies in question are interest rate rules which react to inflation or NGDP. The authors find again that overall NGDP targeting outperforms inflation targeting.

Stability and uniqueness of equilibrium in models where NGDP targeting is used has been a concern about the policy. In Mitra (2003) the author determines the determinacy of the steady-state and E-stability of the rational expectations equilibrium (REE) in a simple NK model with NGDP growth targeting. He finds that NGDP growth targeting results in a unique steady-state and the REE is E-Stable when contemporaneous growth is targeted. The author does find that targeting expected NGDP growth leads to indeterminacy and the minimum state variable solution is not E-Stable.

Honkapohja and Mitra (2014) compares an interest rate rule that targets NGDP to an interest rate rule that targets the price level and the output gap. All rules are subject to the zero lower bound, the model is a standard NK model, and agents are assumed to make forecasts using infinite horizon learning of the steady-state values of output, inflation, and the nominal interest rate. They prove that both rules have expectationally stable targeted steady states when the price adjustment costs are sufficiently small and they prove that for both rules the zero lower bound constrained steady state is not expectationally stable. Comparing interest rate rules with a quadratic loss function they find that when agents take the central bank's guidance of the particular rule into account both rules outperform a simple Taylor rule and the NGDP rule outperforms the price level

rule. Without the guidance the rules induce a smaller basin of attraction where expectations converge to the intended steady state than a Taylor rule.

This paper is also related to research on financial frictions. Cúrdia and Woodford (2016) develops the model used in this paper. It is essentially a standard NK model with two types of agents and financial frictions. They find that the presence of financial frictions leads to substantial deviations in the responses of endogenous variables to a variety of shocks. The authors then derive a quadratic loss function from household ex-ante utility. They find that in a special case the optimal target is similar to that of the standard NK model, and that the interest rate rule that implement the policy responds to financial frictions and the interest rate spread. To add the authors stress that despite the presence of a credit spread and financial frictions do not lead to many differences in the optimal policy response from standard models.

Eggertsson and Krugman (2012) provides a different look at a New Keynesian model with debt. They assume that there are patient and impatient households. The impatient households borrow up to a constraint which is assumed to be less than the present discounted value of their future income. They find that when agents are debt constrained it gives rise to a variety of phenomena including: Fisherian debt deflation, liquidity traps, backward sloping AD curves leading to the paradoxes of thrift, toil and flexibility arising<sup>1</sup>, a Keynesian multiplier and support for expansionary fiscal policy. They find that for a sufficiently large debt-overhang a contraction in borrowing limits leads to a negative natural rate of interest.

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<sup>1</sup>The paradox of thrift is well known, the paradox of toil refers to increases in the labor supply leading to lower real output, and the paradox of flexibility refers to flexible prices actually leading to less output.

### 3.3 Model

Our analysis uses the linearized model derived in Cúrdia and Woodford (2016). They assume that there are two types of infinitely lived households, ones who are patient and net savers and households who are impatient and net borrowers. Each member of the household supplies a differentiated labor input and earn a wage for it. Households can deposit funds in a bank and earn a return  $1 + i_t^d$  and borrow at a higher rate. The households get utility from consumption and disutility from labor. They choose consumption, labor supply, and whether to save or borrow in order to maximize expected lifetime utility. The impatient households have a higher marginal utility of consumption than the patient ones. A fraction  $\pi_b$  of households are impatient and every period a fraction  $\delta$  of all households redraw their type each period.

There is a banking sector which collects deposits from the patient households and makes loans to impatient households. They earn a return on the loans and pay out interest on the deposits. Financial frictions arise because real resources are used in the production of loans and some loans are fraudulent. These frictions contribute to the spread between the interest rate earned by savers and the rate paid by borrowers. The real cost of producing loans is a function of both an exogenous component and the current real quantity of loans.

Finally the product sector is in the same form as a standard NK model. Intermediate goods producing firms with market power face a probability  $\alpha$  that they will be able to reset their wage every period. This gives rise to a Phillips curve that looks similar to the standard one in the literature.

The endogenous variables in this model are  $i_t^d$ , the depositor's interest rate;  $Y_t$ , output;  $\pi_t$ , inflation,  $\Omega_t$ , the ratio of marginal utilities of consumption the two

types of households;  $\lambda_t$ , the average of the marginal utility of consumption the two types of households;  $\omega_t$ , the spread between the rate depositors are paid and the rate borrowers pay; and  $b_t$ , the level of private debt. The exogenous variables are  $G_t$ , government spending;  $\bar{c}_t^h$ , the exogenous shock to household  $h$ 's marginal utility of consumption;  $\Xi_t$ , shocks to the cost of producing loans;  $\chi_t$ , the amount of fraudulent loans;  $\tau_t$ , subsidies to producers;  $\mu_t^w$ , the wage markup; and  $b_t^g$ , government borrowing.

Log linearizing each household's Euler equation and defining  $\Omega_t$  as the difference in log deviation from steady state between the borrower and the saver's marginal utility then we have that:

$$\Omega_t = \hat{\delta} E_t \Omega_{t+1} + \omega_t, \quad (3.1)$$

where  $\omega_t$  is the log deviation from the steady state of the spread between the rate borrowers pay on loans and the rate paid to depositors.  $\hat{\delta}$  is a function of the subjective discount factor, the steady state real interest rate paid to depositors, the steady state spread, and the probability of having your preferences reset and the relative proportions of the household types. In (3.1) we see that this ratio is increasing with expectations of next period's ratio and the spread between the two interest rates. When the spread increases only the borrower's Euler equation is affected. The increased spread lowers consumption and increases the marginal utility of the borrowers increasing the difference in the marginal utilities.

Additionally the two household type's Euler equations can be combined as a weighted average and plugged into aggregate demand to produce something that looks closer to the traditional New Keynesian IS curve

$$Y_t = E_t Y_{t+1} - \bar{\sigma}(i_t^{avg} - E_t \pi_{t+1}) - E_t \Delta g_{t+1} - E_t \Delta \Xi_{t+1} - \eta s_{\Xi} E_t \Delta b_{t+1} - \bar{\sigma} s_{\Omega} \Omega_t + \bar{\sigma}(s_{\Omega} + \psi_{\Omega}) E_t \Omega_{t+1}, \quad (3.2)$$

where  $Y_t$  is the log deviation from steady state of output,  $i_t^{avg}$  is the average between the interest rate paid by the borrowers and the interest rate paid to depositors it is equal to  $i_t^d + \pi_b \omega_t$ ,  $\pi_{t+1}$  is inflation. The variable  $\Xi_t$  is defined as:

$$\Xi_t \equiv \frac{\bar{b}^{\eta}}{\bar{Y}} (\tilde{\Xi}_t - \bar{\Xi}_t),$$

where  $\tilde{\Xi}_t$  is the exogenous component for the real cost of loans,  $\bar{b}$  and  $\bar{Y}$  are the steady state values of real private loans and output respectively, and  $\eta$  determines the contribution of the level of real loans to the cost of producing loans. When  $\eta = 0$  the level of loans do not contribute to the cost of producing loans. The variable  $b_t$  is the current level of real private debt and  $s_{\Xi}$  is the steady state share of output eaten up by the loan production costs.

The variable  $g_t$  captures shocks to aggregate expenditures. It is defined as:

$$g_t = s_c \bar{c}_t + G_t,$$

where  $s_c \bar{c}_t$  is the weighted average of the log deviations of the exogenous disturbances to marginal utility of the two types,  $\bar{c}_t^b$  and  $\bar{c}_t^s$ , multiplied by the steady state consumption share of each type, and  $G_t$  is the deviation from steady state of government spending as a percentage of steady state output.

$\Delta X_t$  is the first difference of the variable  $X_t$ . The constant  $\bar{\sigma}$  is a weighted average of the two households CRRA utility parameters. The constant  $s_{\Omega}$  is a function of relative proportions of the two household types, the share of output of

each type's consumption, and their CRRA utility parameters. Finally  $\psi_\Omega$  depends on the same parameters as  $\hat{\delta}$ .

The Phillips curve is derived in usual fashion linearizing the profit maximizing first order condition for firms resetting their price and linearizing the evolution of the price level:

$$\pi_t = \beta E_t \pi_{t+1} + u_t + \kappa(Y_t - Y_t^n) - \xi \bar{\sigma}^{-1}(\Xi_t + \eta s_\Xi b_t) + \xi(s_\Omega + \pi_b - \gamma_b)\Omega_t, \quad (3.3)$$

where  $\beta$  is the subjective discount factor,  $\xi$  is a function of the reset probability, the subjective discount factor, the elasticity of output with respect to labor, the Frisch elasticity of labor, and the CES parameter for the differentiated goods produced by the monopolistically competitive firms. The parameter  $\kappa$  is:

$$\kappa \equiv \xi(\phi(1 + \nu) - 1 + \bar{\sigma}^{-1}),$$

where  $1/\phi$  is the output elasticity with respect to labor,  $1/\nu$  is the Frisch elasticity of labor supply. The constant  $\gamma_b$  in (3.3) depends upon the relative proportions of borrowers and savers, the ratio of the borrower's marginal utility to the average marginal utility, and the scalars multiplying the disutility of labor in the objective function of the household.

The variable  $u_t$  is a linear combination of  $\mu_t^w$  and  $\tau_t$ , which are the log deviation from the steady state of the wage markup and the negative of the log deviation from the steady state of the untaxed fraction of firm revenues. Finally,  $Y_t^n$  is the natural level of output in log deviations from steady state form. It is a linear combination of  $\bar{h}_t$ , the labor disutility preference shifter in deviation



from steady state form,  $z_t$ , the log deviations from the steady state of aggregate productivity, and  $g_t$ .

The log linearized equation governing the interest rate spread is obtained by log linearizing the first order condition for the financial intermediaries:

$$\omega_t = \omega_b b_t + \omega_\chi \chi_t + \omega_\Xi \Xi_t, \quad (3.4)$$

where  $b_t$  is the level of real private borrowing in log deviation from steady state form and  $\chi_t$  is the deviation of the exogenous component of the quantity of fraudulent loans from its steady state value multiplied by a term involving steady state debt and a parameter governing the contribution of the debt level towards fraud. The coefficients are functions of steady state debt, interest rate spread, debt to GDP, loan production costs, and fraud.

Log linearizing the evolution of private debt and making some substitutions leads to:

$$\begin{aligned} b_t = & \tilde{\kappa}_1 \bar{c}_t^b + \tilde{\kappa}_2 \bar{c}_t^s + \tilde{\kappa}_3 \lambda_b^t + \tilde{\kappa}_4 \lambda_t^s + \tilde{\kappa}_5 \Omega_t + \tilde{\kappa}_6 \mu_t^w + \tilde{\kappa}_7 \bar{h}_t + \tilde{\kappa}_8 Y_t + \tilde{\kappa}_9 z_t + \\ & \tilde{\kappa}_{10} \omega_t + \tilde{\kappa}_{11} (i_{t-1}^d - \pi_t) + \tilde{\kappa}_{12} (b_{t-1} + \omega_{t-1}) + \tilde{\kappa}_{13} (b_t^g - \delta(1 + \bar{r}^d) b_{t-1}^g) + \tilde{\kappa}_{14} (G_t + \Xi_t), \end{aligned} \quad (3.5)$$

where the  $\tilde{\kappa}$  terms are functions of model parameters and steady state values of model variables. The variable  $b_t^g$  refers to government debt at time  $t$ .

### 3.4 Monetary Policy

We will compare four different NGDP based monetary policies with four conventional monetary policies and the optimal policy for the model. The key

policies of interest are ones that involve targeting the level or the growth of nominal GDP. We consider interest rate rules that respond to deviations of nominal GDP growth from the growth of the natural level of output and the targeted inflation rate and the deviations of nominal GDP level from the natural level of output and the targeted price level plus some random policy error. In addition to these two, we also consider the case where monetary policymakers can perfectly hit their targets for NGDP growth or level plus a mean zero random disturbance.

The traditional policies considered are two Taylor rules, inflation targeting, and a flexible inflation target.

For the interest rate rules considered the central bank adjusts the money supply to achieve a target for  $i_t^d$ . One rule we consider (equation 3.6) adjusts the interest rate based on deviations of nominal GDP growth,  $\pi_t + Y_t - Y_{t-1}$ , from the bank's target for it,  $Y_t^n - Y_{t-1}^n$ , which is the growth in the natural level of output and we assume the targeting rate of inflation is zero.

$$i_t^d = \varphi(\pi_t + (Y_t - Y_{t-1}) - (Y_t^n - Y_{t-1}^n)) + \epsilon_t \quad (3.6)$$

We also look at an interest rate rule that adjusts based upon deviations of the level of NGDP (equation 3.7),  $p_t + Y_t$ , from a targeted level of NGDP equal to  $Y_t^n$ . Here  $p_t$  is the log price level and the targeted price level is normalized to be equal to one.

$$i_t^d = \varphi(p_t + Y_t - Y_t^n) + \epsilon_t \quad (3.7)$$

In addition we look at standard versions of the Taylor rule for comparison purposes. Equations (3.8) and (3.9) are standard Taylor rules where output is targeted to either equal its steady state value or its natural level.

$$i_t^d = \varphi_\pi \pi_t + \varphi_Y (Y_t - Y_t^n) + \epsilon_t \quad (3.8)$$

$$i_t^d = \varphi_\pi \pi_t + \varphi_Y Y_t + \epsilon_t \quad (3.9)$$

In addition to the interest rate rules we also consider the case where the central bank always achieves its target, under these rules the bank adjusts their preferred policy instrument (in the model this comes in the form of the deposit rate) until it achieves the desired target. With two exceptions these policies are assumed to be implemented with error. The first of these policies (equation 3.10) is a target for NGDP growth, where policy adjusts to set it exactly equal to the growth rate of the natural level of GDP.

$$\pi_t + Y_t - Y_{t-1} - \epsilon_t = Y_t^n - Y_{t-1}^n \quad (3.10)$$

In addition we consider a policy (equation 3.11) where the central bank attempts to set the level of NGDP  $p_t + Y_t$  equal to the natural level of output.

$$p_t + Y_t - \epsilon_t = Y_t^n \quad (3.11)$$

We also consider an inflation target (equation 3.12) where the central bank conducts monetary policy in order to achieve zero percent inflation (with error).

$$\pi_t + \epsilon_t = 0 \quad (3.12)$$

The target in (3.13) is a flexible inflation target and the optimal target in a simple NK model derived from the first order conditions associated with minimizing the welfare loss function derived from the quadratic approximation to the ex-ante

household utility.

$$\pi_t + \lambda_x(y_t - y_{t-1}) = 0, \tag{3.13}$$

$$y_t = Y_t - Y_t^*,$$

$$Y_t^* = Y_t^n + \frac{1}{\omega_y \bar{\sigma} + 1} \Xi_t.$$

With rules in the form of (3.10-3.13) concerns of practicality can arise. Traditionally in the literature, it is assumed that the central bank follows an interest rate rule that is conditioned on exogenous variables and lags and expectations of endogenous variables. Due to the complexity of this system the expressions for these interest rate rules are long and complex leaving little intuitive interpretation so we have chosen not to reproduce them. Be aware however that in the background something like one of these rules is being implemented or the central bank is behaving as if they are carrying out an interest rate rule that achieves its target.

We will evaluate the rules using welfare calculations based on a second order approximation to the ex-ante utility function for the representative household, impulse response functions, and the unconditional variance of key endogenous variables.

### 3.5 Results

We find that NGDP based policies usually beat out the traditional policies of Taylor rules and inflation targets. None of these policies outperform the flexible inflation target in this model. To understand why NGDP level policies and most NGDP growth policies outperform most of the traditional policies we will examine impulse response functions to show how the NGDP policies typically remain close

to the flexible inflation target impulse response functions even when (3.8), (3.9) and (3.12) deviate. Then to further understand the source of the welfare gains we will present key unconditional variance and covariances for these rules.

Our calibration is the same as the one used in Cúrdia and Woodford (2016), we set baseline  $\varphi$  for both the NGDP growth and level rule equal to 2 to start. We will also present welfare results for interest rate rules where the response coefficients are selected between zero and twenty to maximize welfare. Welfare is calculated using a second order approximation of the ex-ante utility of a representative household. Using the computed welfare we calculate the percentage increase in consumption necessary to make the household indifferent between the policy in question and the optimal policy derived using the linear quadratic approximation. Since there are two types of households we present three scenarios. In the first only borrower's consumption is increased. In the second only saver's consumption is increased. Finally, we present the case where both types of household's consumption are increased by the same percentage. These ultimately present the same ordering of the different policies in terms of the preference of a household's lifetime utility. The relative size of the increase needed is larger for savers because their marginal utility of consumption is always lower for a given level of consumption. The relative percentage increase for both is smaller than the others because it is boosting aggregate consumption.

Because flexible inflation targeting is the optimal policy under a typical representative agent NK model and the particular flexible inflation target is calibrated to be the optimal policy under no financial frictions in this model with a weight on deviations in real output growth from its target,  $\lambda_x$ , is equal to 0.1256. It performs almost as well as the optimal policy.

TABLE 2. Consumption Equivalent Welfare

Policy	Borrower	Saver	Both
Flexible Inflation Target	0.002	0.003	0.001
Optimal NGDP Level Rule	0.004	0.006	0.002
NGDP Level Rule	0.004	0.006	0.003
NGDP Level Target	0.007	0.011	0.004
Optimal Taylor Output Gap Rule	0.008	0.012	0.005
Optimal NGDP Growth Rule	0.010	0.016	0.006
NGDP Growth Target	0.011	0.017	0.007
Inflation Targeting	0.030	0.052	0.019
Optimal Taylor Rule	0.046	0.095	0.031
Taylor Gap Rule	0.063	0.130	0.042
Taylor Rule	0.069	0.142	0.046
NGDP Growth Rule	0.550	1.019	0.345

The next best is the interest rate rule that responds to deviations in NGDP's level from its target with an optimized response parameter set at  $\varphi = 1.5074$ . This is not too far from the calibration we set for the unoptimized rule,  $\varphi = 2$  which falls a bit behind it in terms of welfare performance. The level target is behind both of those yet still above any traditional policies.

The NGDP growth based policies do not appear to be preferable to level based policies. The unoptimized interest rate rule performs the worst amongst any of the policies. The optimized interest rate rule sets  $\varphi = 20$  and along with the growth based target is in the middle of the pack amongst policies considered.

A Taylor rule which responds to deviations in real output from its natural level with optimized weights  $\varphi_\pi = 19.9975$  and  $\varphi_Y = 3.0489$  is the best amongst traditional monetary policy. Inflation targeting is below most of the NGDP based policies, as are the optimized Taylor rule that doesn't respond to the output gap

with  $\varphi_\pi = 8.007$  and  $\varphi_Y = 0.049945$ , the unoptimized Taylor rules were set equal to  $\varphi_\pi = 2$  and  $\varphi_Y = .25$  for comparison.

While the flexible target requires less extra consumption to leave the representative household indifferent to the optimal policy, it is not that much closer in consumption terms than some of the NGDP based policies. In addition one could argue that the NGDP based policies would be easier for policymakers to achieve as the flexible inflation target here involves estimating structural relationships in the economy, which could be tenuous, and also estimating not just the natural level of output but the degree of financial frictions. In addition, an NGDP target would appear to be easier for the public to understand when forming expectations of future policy than the flexible inflation target.

### Impulse Response Functions

We now discuss impulse response functions (IRF) to provide an explanation for why NGDP policies perform so well overall, in particular, the level based policies. We use the IRF for the flexible inflation target as a baseline to compare the other policies. The full suite of IRFs can be found in the appendix.

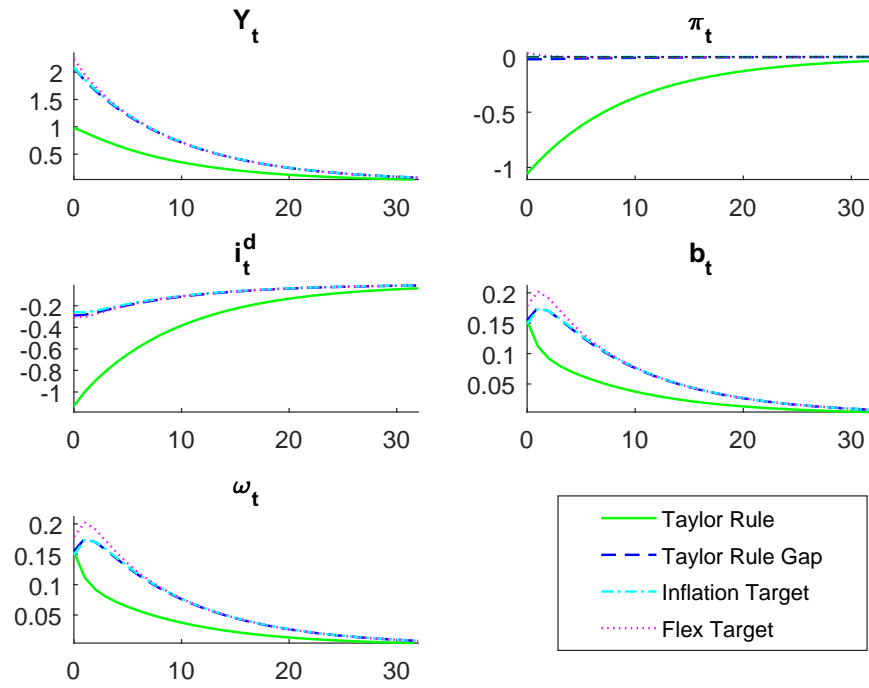
The IRFs for productivity shocks give us our first look at why the NGDP growth interest rate rule performs so poorly. The IRFs for the NGDP policies are presented in figure 11b where we can see that the NGDP growth interest rate rule's responses are sometimes closer to the responses generated by a simple Taylor rule as presented in figure 11a. Most policies generated responses that were close to the flexible inflation target. The exact NGDP targets are able to very closely track the flexible inflation targets as well. The preferred responses to productivity shocks feature small decreases of the depositor interest rate which gradually fades out, inflation slightly increasing and the fading out quickly, a small increase in the

interest rate spread and the level of private debt which further increase and die out, and a large increase in output which dies out gradually. The NGDP growth based interest rate rule features IRFs closer to the Taylor rule which also features a larger decrease in the depositor's interest rate and some deflation. For both rules output will consistently fall below the natural level which leads to deflation. Under labor supply shocks which can be found in the appendix (Figs. 24a and 24b) responses feature similar responses given that the change in preferences leads to a smaller increase in the natural level of output.

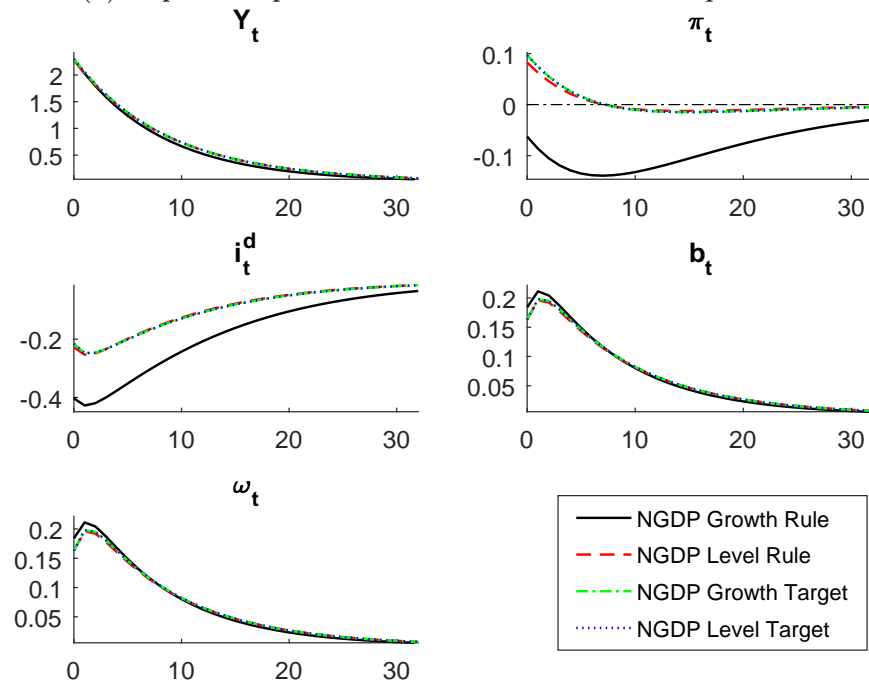
The impulse response functions for increases in the costs of loan production and the quantity of fraudulent loans are on display in figures 12 and 13 respectively. As with productivity shocks the exact targets result in responses very similar to those under flexible inflation targeting, and the NGDP level interest rate rule also results in similar IRFs as well. The growth rate rule and the other traditional monetary policies feature larger and longer drops in output and inflation in response to these financial frictions shocks. All policies lead to large drops in interest rates, increases in spreads, and drops in private borrowing.

Responses to an increase of  $\tau_t$  which implies an increase in after-tax revenues by the firms, these are on display in figure 14. The responses are identical to responses to  $\mu_t^w$ , which can be found in the appendix in figure 25. These shocks have an inflationary effect in (3.3), the flexible inflation target responds to the increase in inflation with a decrease in output level initially and some negative output growth following that along with some positive inflation. Eventually, output growth becomes positive and there is a small spell of deflation. The exact targets for NGDP level and growth replicate the general path of these two variables but because output and inflation (price level) are weighted equally under these targets the initial response of output is smaller and the response of inflation is larger. The



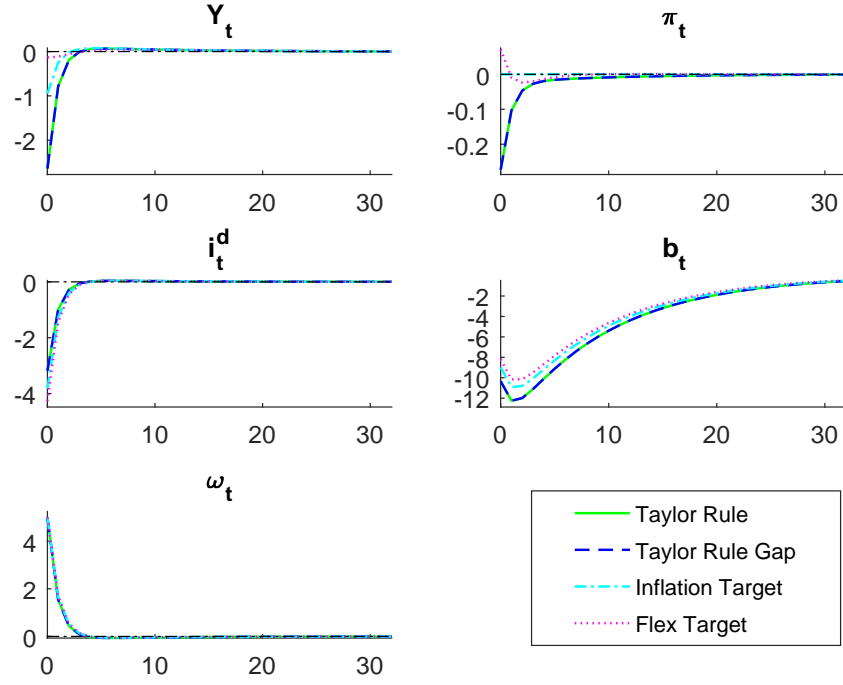


(a) Impulse response functions under traditional policies.

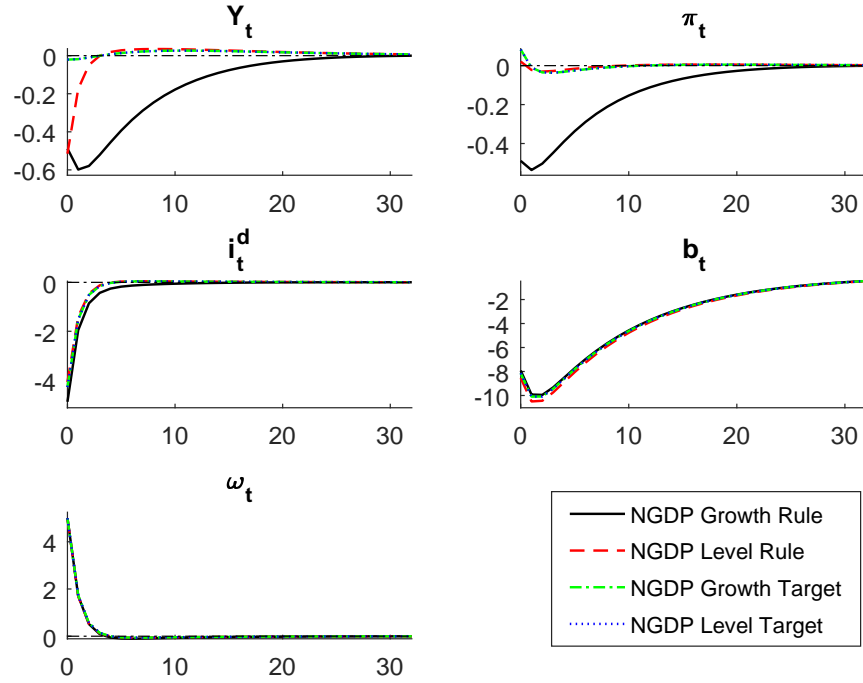


(b) Impulse response functions under NGDP based policies.

FIGURE 11. Impulse response functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to productivity shocks.

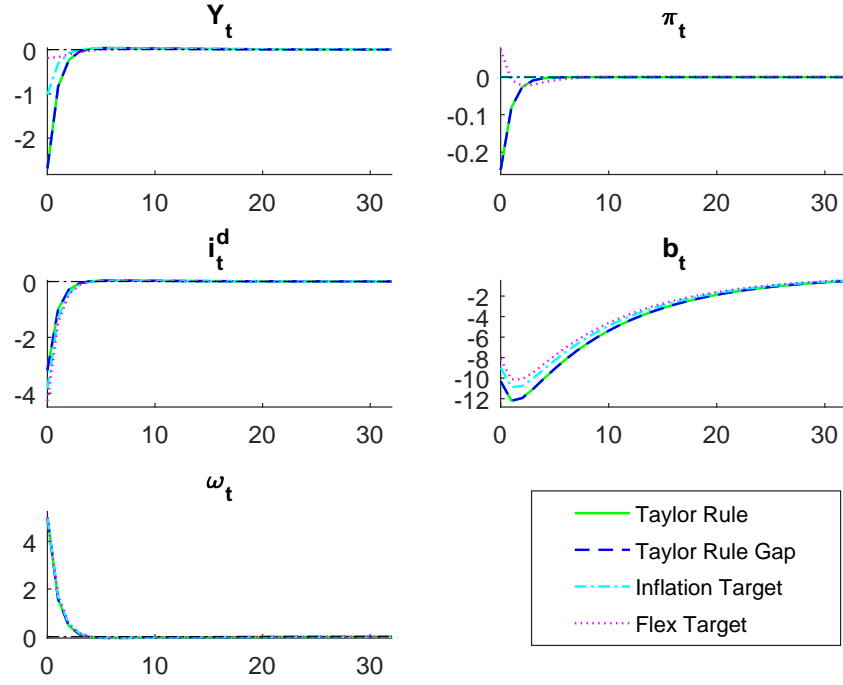


(a) Impulse response functions under traditional policies.

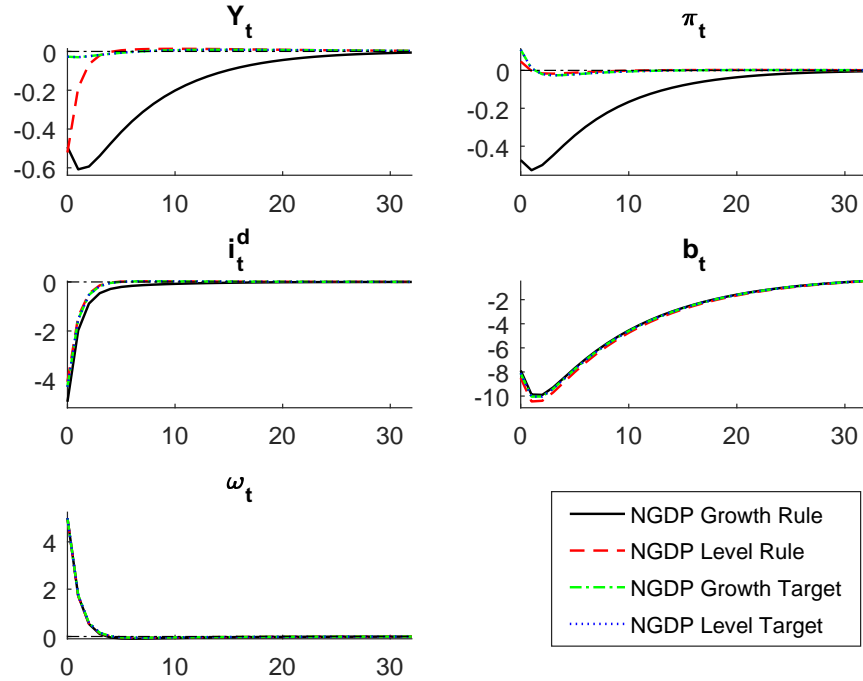


(b) Impulse response functions under NGDP based policies.

FIGURE 12. Impulse response functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to loan cost shocks.



(a) Impulse response functions under traditional policies.



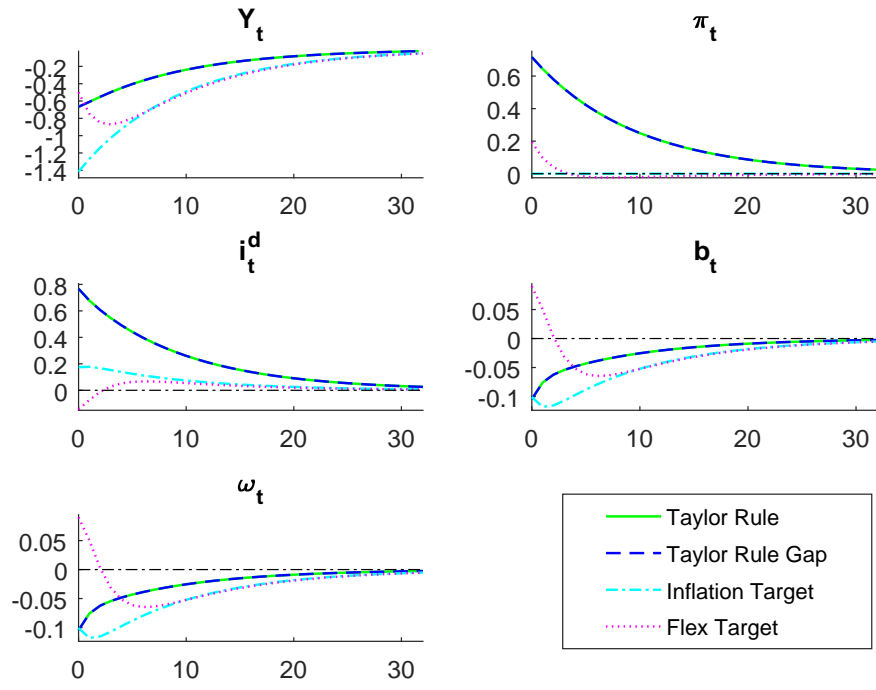
(b) Impulse response functions under NGDP based policies.

FIGURE 13. Impulse response functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to shocks to the amount of fraudulent loans.

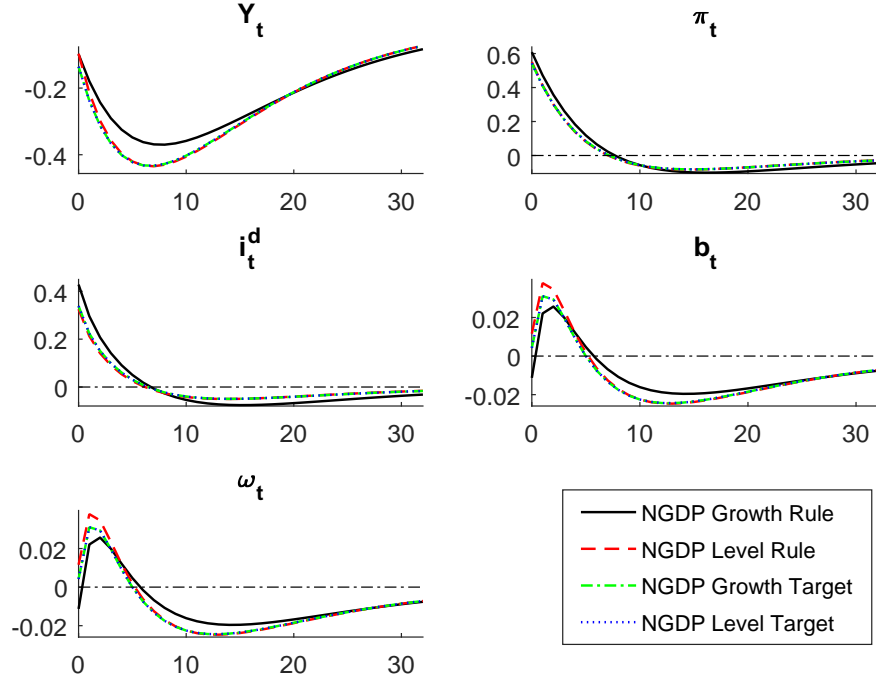
interest rate rules also track these movements fairly closely. The main deviation between these rules and the flexible target is that the depositor interest rate actually falls under a flexible inflation target while the interest rate spread and private borrowing increase and fall by much more compared to the NGDP policies. The more traditional policies match the flexible inflation target less well to varying degrees.

The IRFs in response to government borrowing shocks (Fig. 15) look very similar with respect to  $i_t^d$ ,  $b_t$ , and  $\omega_t$ , but deviate in the response of output and inflation. The policies that featured the lowest welfare loss relative to the optimal policy featured small increases in output which fade out quickly and small fluctuations in inflation. The traditional policies feature larger increases in output than any of the other policies and a small spike in inflation that quickly fades out. The NGDP growth interest rate rule features a smaller increase in output which is followed by further increases in output before gradually returning to steady state and inflation follows a similar path.

Responses to policy error also differ widely across policies (Figs. 16a and 16b). When there is a contractionary policy error under NGDP level targeting only the current value of the policy error is pushing the NGDP level away from its target. Under NGDP growth targeting however these errors accumulate and temporarily push output even further below the initial drop. Notice that with the NGDP level target and interest rate rule and the NGDP growth target and interest rate rule that the IRFs are similar but not exactly the same. Part of this is because an error in choosing an interest rate to set would have a different impact than an error in choosing the target value for NGDP level or growth.

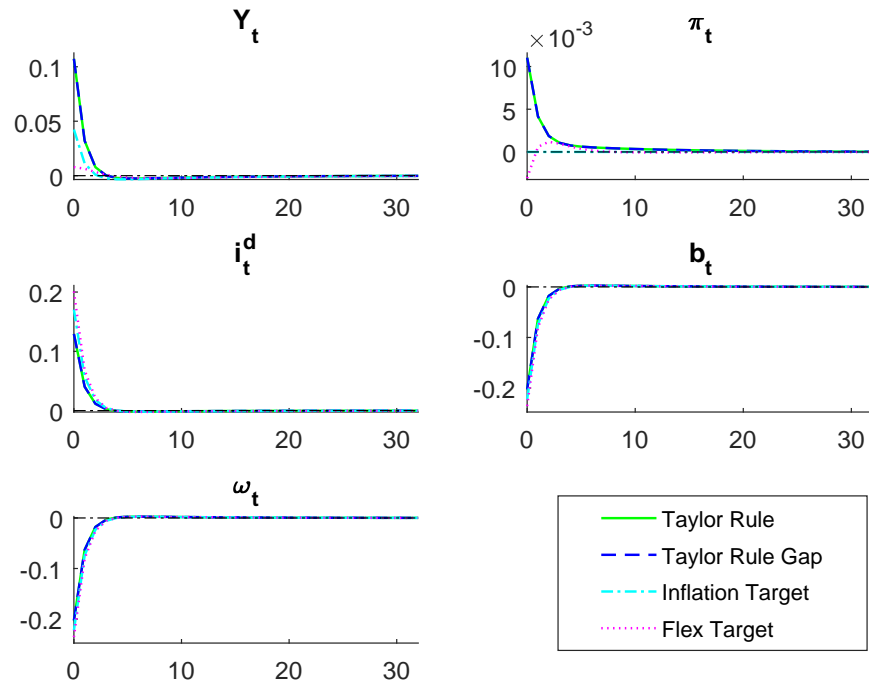


(a) Impulse response functions under traditional policies.

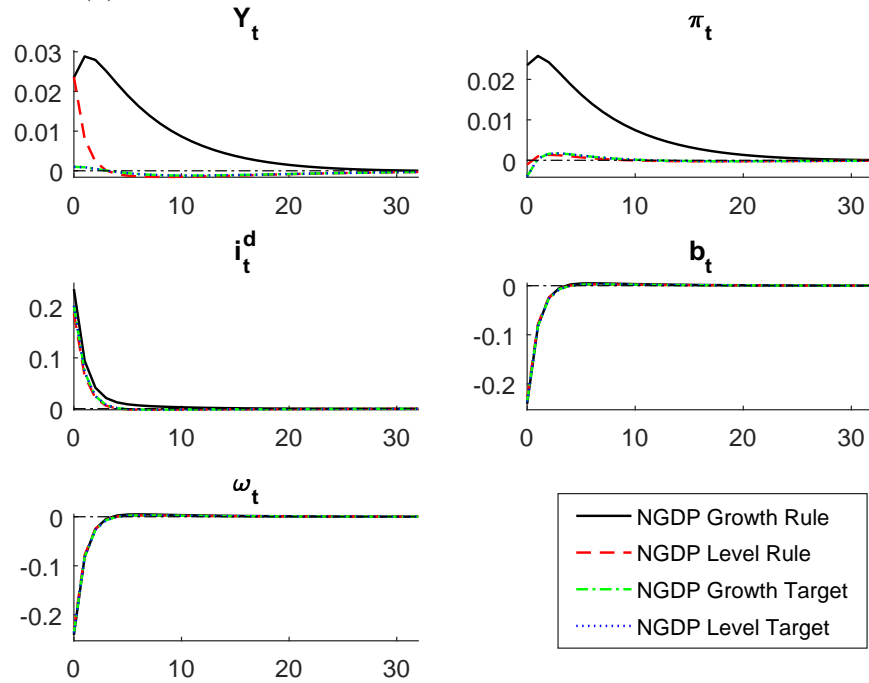


(b) Impulse response functions under NGDP based policies.

FIGURE 14. Impulse response functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to tax shocks.

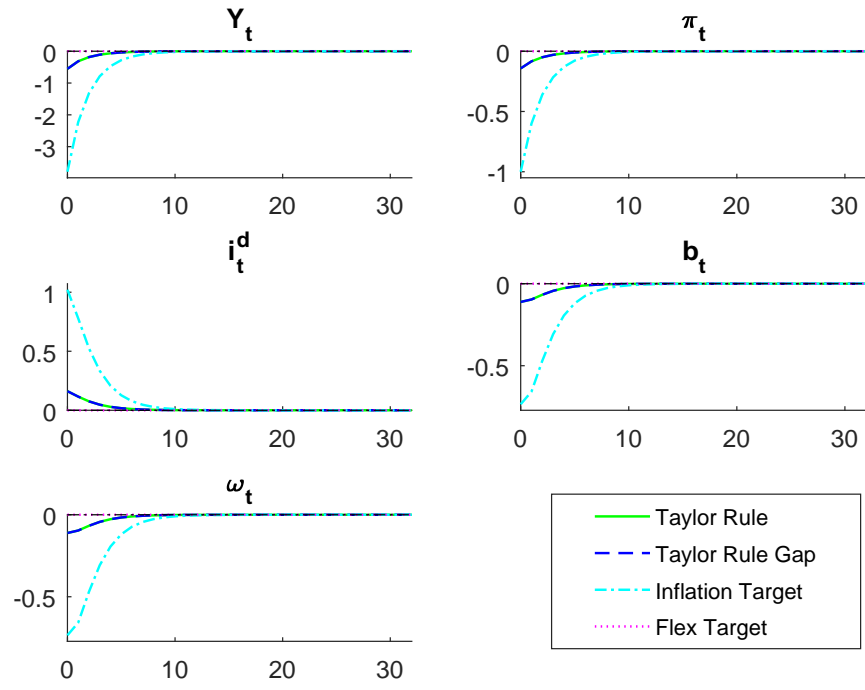


(a) Impulse response functions under traditional policies.

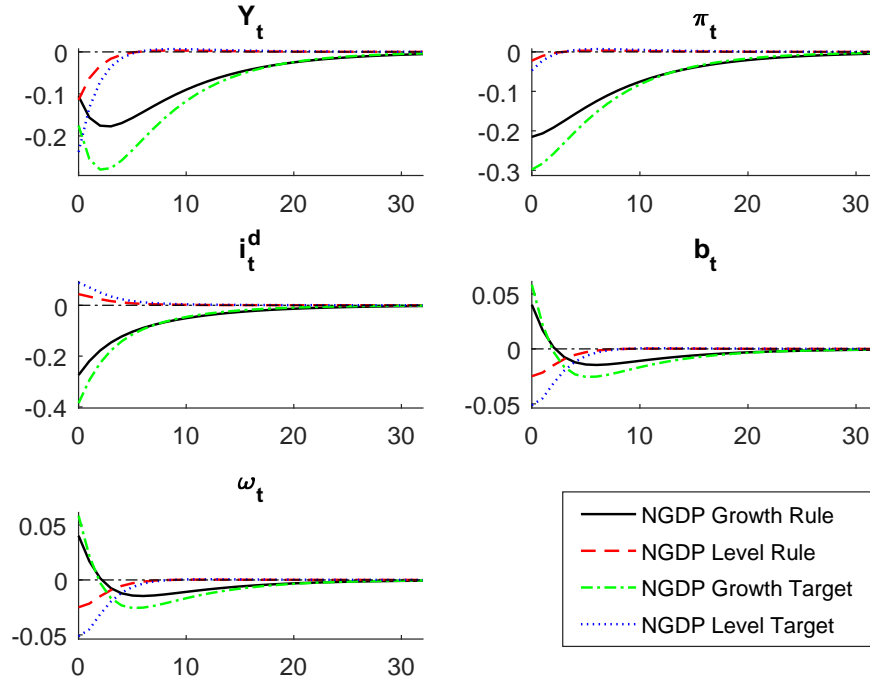


(b) Impulse response functions under NGDP based policies.

FIGURE 15. Impulse response functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to shocks to the level of public borrowing.



(a) Impulse response functions under traditional policies.



(b) Impulse response functions under NGDP based policies.

FIGURE 16. Impulse response functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to policy error.

## Unconditional Variance

We use the REE associated with each policy to find the unconditional variance of key variables. These variances are displayed in Tables 3 to 15. We will use this section to discuss sources of the welfare differences across the different policies.

First we compare the NGDP level based policies (Tabs 8, 10, and 12) to the optimal policy (3) and the flexible inflation target (4). What sticks out most is how low the variance of inflation is for the optimal policy. The flexible inflation target is the closest to the variance to the optimal policy. The NGDP level based policies all feature a variance of inflation among the closest to the optimal policy and the flexible inflation target. In addition to inflation, the NGDP level targets all deliver lower output variance than most other policies, including the optimal policy. The variance of private debt also slightly lower across NGDP level targets compared to the optimal policy. The variance of the interest rate spread and the interest rate paid to depositors have variances that are similar across all five policies.

With the NGDP growth based policies (Tabs 7, 9, and 11) however in some cases there are large deviations from the variances that prevail under the optimal policy. The exact target for NGDP growth leads to a higher variance of inflation than any of the level based policies, although it is still fairly small. The optimal NGDP growth interest rate rule actually features lower inflation variance than an exact NGDP level target. But the unoptimized NGDP growth interest rate rule features the highest variance of inflation for any policy considered. The growth-based interest rate rule also results in higher variance of output, interest rate spread, and depositor interest rate.



As far as the conventional policies the Taylor rules and the inflation target can be found in Tables 5-6 and 13-15. The unoptimized Taylor rules feature high variance of inflation and output relative to most of the other policies. The optimized Taylor rule still features higher inflation and output variance, but the optimized Taylor gap rule brings inflation and output variance to values that are close to most of the NGDP based policies. The inflation target features somewhat larger inflation variance due to policy errors directly impacting inflation whereas under the NGDP targets policy errors can be spread across inflation and real output.

While the optimal policy minimizes inflation across all rules under consideration, the NGDP level target minimizes output variance, the optimal Taylor output gap rule has the lowest variance of private debt with the NGDP level and growth targets close behind, the variance of the interest rate spread and the depositor rate are smallest under the optimal Taylor rule.

### The Role of Debt

A natural question would be what role debt actually plays in the desirability of the NGDP policies. One way to explore this is by looking at the welfare results for a similar representative agent model. The results are displayed in Table 16. The most striking difference is that with a representative agent inflation targeting becomes the second most preferred choice to flexible inflation targeting. This might imply that the presence of debt is driving our results, however running the model with borrowing but no financial frictions yields the same outcome.

TABLE 3. Optimal Policy  
Unconditional Variances

Inflation	0.0594
Output	149.9985
Private Debt	13087.1013
Spread	188.8153
Depositor Rate	136.7203

TABLE 5. Taylor Rule  
Unconditional Variances

Inflation	2.6500
Output	1004.9558
Private Debt	13135.0834
Spread	185.1502
Depositor Rate	111.9412

TABLE 7. NGDP Growth  
Target  
Unconditional Variances

Inflation	0.8666
Output	50.0522
Private Debt	12985.3281
Spread	194.2096
Depositor Rate	144.0994

TABLE 9. NGDP Growth  
Interest Rate Rule  
Unconditional Variances

Inflation	24.7492
Output	410.1599
Private Debt	13051.7203
Spread	206.0169
Depositor Rate	263.9703

TABLE 11. Optimal NGDP  
Growth  
Interest Rate Rule  
Unconditional Variances

Inflation	0.5976
Output	40.8073
Private Debt	12992.8962
Spread	194.9780
Depositor Rate	150.6716

TABLE 4. Flexible Inflation  
Target  
Unconditional Variances

Inflation	0.1425
Output	75.0301
Private Debt	13038.9964
Spread	192.3647
Depositor Rate	139.8009

TABLE 6. Taylor Gap Rule  
Unconditional Variances

Inflation	2.2731
Output	1023.9337
Private Debt	13135.0612
Spread	185.1488
Depositor Rate	111.5488

TABLE 8. NGDP Level Target  
Unconditional Variances

Inflation	0.6533
Output	30.7413
Private Debt	12985.3657
Spread	194.1314
Depositor Rate	142.2278

TABLE 10. NGDP Level  
Interest Rate Rule  
Unconditional Variances

Inflation	0.5101
Output	63.5297
Private Debt	13019.1650
Spread	192.2970
Depositor Rate	134.9671

TABLE 12. Optimal NGDP  
Level  
Interest Rate Rule  
Unconditional Variances

Inflation	0.4909
Output	87.3074
Private Debt	13029.7332
Spread	191.7117
Depositor Rate	132.6492

TABLE 13. Optimal Taylor Rule  
Unconditional Variances

Inflation	1.1409
Output	1072.7638
Private Debt	13145.0611
Spread	183.5256
Depositor Rate	101.3136

TABLE 14. Optimal  
Taylor Gap Rule  
Unconditional Variances

Inflation	0.5678
Output	71.6668
Private Debt	12969.2653
Spread	191.8936
Depositor Rate	130.6403

TABLE 15. Inflation Targeting  
Unconditional Variances

Inflation	1.5625
Output	726.5008
Private Debt	13044.7180
Spread	187.2934
Depositor Rate	110.0239

It is the response to financial frictions that makes NGDP based policies preferable to the traditional policies. Return to Figures 12 and 13 and observe that while inflation targeting mostly delivers similar results to the flexible inflation target it allows real GDP to fall in response to the shock. The explicit NGDP level and inflation targets leave output relatively unchanged and the NGDP interest rate rules leave output relatively unchanged initially compared to the more traditional policies. The NGDP growth interest rate rule does leave output below its steady state for longer than any other policy which might help explain its poor performance overall.

### Determinacy

One final question regarding the desirability of NGDP based policies is whether or not the rational expectations equilibrium (REE) associated with each is determinate. Under the calibration used NGDP level and growth targets lead to

TABLE 16. Consumption Equivalent Welfare for Representative Agent

Policy	Welfare Loss
Flexible Inflation Target	0.000
Inflation Target	0.003
Optimal NGDP	0.050
Level Rule	0.052
NGDP Level Rule	0.054
NGDP Level Target	0.057
Optimal NGDP	0.267
Growth Rule	0.267
Optimal Taylor	0.330
Gap Rule	0.353
Taylor Gap Rule	0.478
NGDP Growth Rule	1.447
Optimal Taylor Rule	
NGDP Growth Target	
Taylor Rule	

TABLE 17. Consumption Equivalent Welfare no Financial Frictions

Policy	Borrower	Saver	Both
Flexible Inflation Target	0.000	0.000	0.000
Inflation Target	0.002	0.004	0.001
Optimal NGDP	0.049	0.071	0.029
Level Rule	0.051	0.074	0.030
NGDP Level Rule	0.051	0.074	0.030
Optimal NGDP	0.053	0.077	0.031
Growth Rule	0.254	0.392	0.152
NGDP Level Target	0.254	0.392	0.152
Optimal Taylor	0.316	0.480	0.187
Gap Rule	0.450	1.174	0.315
Taylor Gap Rule	0.455	0.740	0.274
NGDP Growth Rule	1.362	1.973	0.761
Optimal Taylor Rule			
NGDP Growth Target			
Taylor Rule			

determinate REEs. Further exploration of determinacy as well as E-stability in an NK model under NGDP growth targets can be found in Mitra (2003).

As for the interest rate rules, under the calibration used in this paper we find that an NGDP growth interest rate rule leads to a determinate REE when  $\varphi$  is strictly greater than one. For the NGDP level interest rate rule determinacy occurs when  $\varphi$  is strictly greater than zero. This is in line with the results presented in Brennan (2018b).

### 3.6 Conclusion

We have used an NK model with financial frictions and heterogeneous preferences across households to examine the desirability of a variety of NGDP targeting based rules. The model used featured fluctuations in interest rate spreads due to loan production costs, fraud, and the total amount of lending. We looked at the welfare calculations across different types of policies and found that outside of the optimal policy and a flexible inflation target the NGDP based policies generally performed better than other policies considered. We used the IRFs and unconditional variances under these policies to provide an explanation for why the NGDP based policies performed so well and in the case of an NGDP growth interest rate rule why it performed so poorly. We found that the source of these results was the presence of financial frictions as a model without financial frictions and the representative agent version of the model preferred inflation targeting above all NGDP based policies.

These results suggest that an NGDP based policy and in particular a rule of thumb for the policy rate that responds to deviations of the level of NGDP from its target might be preferable to current policy. One avenue for future research would be to alter different aspects of the model and see if the results still hold.

One possible alterations would be to see if the results still hold on a Bernanke-Gertler-Gilchrist financial frictions model or if these results are unique to household borrowing, allowing for trend inflation, and possibly incorporating wage stickiness, habit persistence, and capital. Another possible change is allowing for the impact of policy changes to occur with a lag.

## CHAPTER IV

### NGDP TARGETING, TREND INFLATION, DETERMINACY, AND E-STABILITY

#### 4.1 Introduction

For decades many central banks have chosen to target inflation or seem to follow a Taylor rule. The rationale for choosing these types of policies was that they would help to keep inflation under control and also to minimize the volatility of the output gap. The Great Recession showed that there are limitations to these policies during severe recessions. Nominal GDP (NGDP) targeting has been proposed as a policy that would be able to prevent severe recessions and as a policy that puts stabilizing real output on equal footing with stabilizing the price level.

One of the key questions about the desirability of NGDP targeting is whether it will keep expectations under control. By this we mean will it result in a determinate and E-stable rational expectations equilibrium (REE). This question has been answered by Mitra (2003) and Brennan (2018b) for the zero trend inflation case, in this paper we will present results for non-zero trend inflation. Cogley and Sbordone (2008) and Ascari and Ropele (2009) showed that under positive trend inflation properties will depend on the value of trend inflation in a basic New Keynesian (NK) model. Ascari et al. (2017b) and Kurozumi and Van Zandweghe (2017) have shown that positive trend inflation will change the E-stability properties of simple NK models. We will use a standard NK model with non-zero trend inflation and partial indexation of prices to trend inflation and/or past values of inflation and examine the determinacy and E-stability properties of the REE under NGDP targeting interest rate rules.

We are interested in NGDP targeting policies because NGDP has been suggested as an alternative intermediate target for monetary policy. Its equal weighting of output and inflation stabilization, countercyclical debt movements, and desirable zero lower bound properties make it an attractive option.

Brennan (2018b) covers the determinacy and E-stability properties under NGDP targeting policies with no trend inflation, but it is not necessarily the case that these properties still hold when nonzero trend inflation is present. It has been shown, for example in Ascari and Ropele (2009), that the presence of trend inflation can change the normal conditions for E-Stability and determinacy when using a Taylor rule. Therefore the determinacy and E-stability requirements for one of the interest rate rules we are considering may change under non-zero trend inflation when the central bank is pursuing some form of NGDP targeting.

Our key result is that under nonzero trend inflation contemporaneous NGDP level targeting results in determinacy and E-stability for all parameterizations considered. While contemporaneous NGDP growth targets perform well, they require a minimum value for the response parameter needed to yield E-stability and determinacy in most settings. Rules which target the expected values of the level or growth rate of NGDP can have very small regions of the parameter space for which the equilibrium is determinate, E-stable, and covariance stationary. As the CRRA utility parameter decreases the range of determinate and E-stable values also decrease under the expected targets. Indexation serves to increase the maximum value of  $\Pi$  that is allowed in the model, but that appears to be the only effect.

The results in this paper as trend inflation falls to zero become identical to the results found in Brennan (2018b). We build on these results by identifying what levels of positive trend inflation lead to changes in these results, and which policies



result in determinacy and E-stability properties that are robust to positive trend inflation.

The rest of this paper is laid out as follows: Section 2 lays out the literature on trend inflation in NK models and NGDP targeting in NK models and puts our results in context with previous results. Section 3 lays out the linearized model and proposed interest rate rules. Section 4 presents our numerical results. Finally a derivation of the linearized model from first principles can be found in the appendix.

## 4.2 Literature Review

This paper relates to a few different strands of the literature. We build upon the work that has been done on the effects of nonzero trend inflation in a New Keynesian model. Our results also add to the understanding of E-stability and the convergence of least squares learning in linearized monetary DSGE models. Finally we contribute to the literature on the desirability of NGDP targeting.

### Trend Inflation

Nonzero trend inflation has been shown to cause significant changes to NK models. One early example is Ascari (2004) where the author compares Calvo pricing to Taylor staggered pricing under nonzero trend inflation. Under Calvo pricing the impulse response functions and steady-state output values of the model change significantly while under Taylor pricing they do not change much.

As those results imply, nonzero trend inflation leads to changes in the optimal monetary policy in a NK model. Ascari and Ropele (2007) show that as trend inflation increases it becomes difficult for monetary policy to stabilize inflation. This is because the New Keynesian Phillips Curve becomes flatter as firms become

concerned with making sure their prices are keeping up with inflation. As a result the equilibrium is not always determinate under optimal discretionary policy. Under commitment the optimal interest rate response to cost-push shocks becomes more persistent as trend inflation increases. This is due to the NK Phillips Curve becoming more forward-looking as trend inflation increases.

In Amano et al. (2007) the authors explore whether the type of nominal rigidities matter for the effects of nonzero trend inflation in a simple DSGE model. They use traditional Calvo pricing, a truncated Calvo pricing, and Taylor pricing schemes. Their main findings are that all forms of price stickiness lead to reductions in the unconditional mean of output, consumption and employment, as well as an increase in the unconditional mean of inflation. Calvo pricing sees the largest changes as a result of nonzero trend inflation, and other than the unconditional mean of inflation the truncated Calvo pricing leads to larger changes than Taylor pricing.

In ? the authors explore the changes in the Taylor principle as a result of nonzero trend inflation in a New Keynesian model. They find that as trend inflation increases policy-makers who want to target inflation aggressively need to target output aggressively and vice versa. These results stem from the effects of trend inflation on the slope of the Phillips curve and steady state relative price dispersion.

Trend inflation and indexation has been proposed as a possible causal pathway for the persistence of inflation. Cogley and Sbordone (2008) uses a simple NK model with time-varying trend inflation to provide a theoretical motivation for exploring the source of inflation persistence in U.S. data. They find that the data provides support for a NK model with nonzero trend inflation and that there is not much evidence of inflation indexation in pricing.

Ascari et al. (2011a) provide empirical evidence of the effects of trend inflation on equilibrium uniqueness. They show that increases in trend inflation increase the probability of indeterminacy, although this period of indeterminacy is limited to the second half of the 1970s. When the Federal Reserve began to respond to inflation more aggressively the estimated REE became determinate again.

It is not just Calvo vs Taylor pricing that has been compared in the literature. Ascari et al. (2011b) compare the effects of nonzero trend inflation in a New Keynesian (NK) model with Calvo pricing and with Rotemberg adjustment costs by estimating a NK model with each form of pricing friction. They conclude that Calvo pricing provides a better explanation of the data and attribute it to the effects of Rotemberg pricing on aggregate demand.

Another concern about trend inflation in New Keynesian is how it changes the effects of disinflation on output loss. In Ascari and Ropele (2013) the authors use a medium-scale DSGE model to show the effects of trying to disinflate from various levels of trend inflation. They unsurprisingly find that trying to go "cold-turkey" from high levels of trend inflation to no trend inflation leads to sizable drops in output for about a year until output returns to steady-state. They do find that it is the size of the disinflation that is driving this. A drop from 8% to 6% has roughly the same impact on output as a drop from 4% to 2%, but larger drops lead to larger responses in output.

Kobayashi and Muto (2013) present findings on the E-stability of various interest rate rules that are conditioned on endogenous variables. They find that under a rule that responds to contemporaneous variables E-stability and determinacy appear concurrently and the region for coefficients in the rule which yield both is shrinking as trend inflation is increasing. For future variables there is

only a small area that is both determinate and E-stable and at low levels of trend inflation most parameterizations yield E-stability but that begins to close as trend inflation increases. Finally using lagged variables introduces some explosive regions as well as regions with determinacy and E-instability.

The welfare impact of trend inflation is studied in a medium-scale New Keynesian model in Ascari et al. (2015). Increasing trend inflation from 2% to 4% is shown to lead to sizable drops in consumption equivalent welfare. The authors identify the following mechanisms: sticky wages, trend growth in technology, roundaboutness in production, and its interaction with shocks to the marginal efficiency of investment. They also find that higher trend inflation has desirable effects of dampening the response of output and other variables to shocks to the marginal efficiency of investment.

In Branch and Evans (2017) the authors build on the work of Ascari and Ropele to explore the behavior of the economy under trend inflation and imperfect information. One of their key findings is that under a simple Fisher model as long as the Taylor principle holds the REE is E-stable. They find that under certain assumptions about the weight agents put on new inflation data can lead to beliefs that inflation follows a random walk. These beliefs can be persistent leading to high volatility of economic variables. And most importantly increasing the inflation target can lead to random-walk beliefs.

The important factor for E-stability is transparency and credibility.

Other explorations of the properties of New Keynesian models under trend inflation include: Bakhshi et al. (2003), Coibion et al. (2012), Ascari et al. (2017b), Ascari et al. (2017a), Kurozumi and Van Zandweghe (2017) Arias et al. (2017) and Coibion and Gorodnichenko (2011).

## Nominal GDP Targets

The first significant contributions on NGDP targets date back to the 1980s and 1990s with the work of Taylor (1985), McCallum (1987), Dueker (1993), Hall and Mankiw (1994), ?, and Ball (1997). In some sense NGDP targets can be traced back to Friedman's k-percent rule with certain assumptions imposed on the velocity of money.

The earlier strain of the literature was focused on questions of a.) whether economic variables were covariance stationary under an NGDP target (Ball (1997) and Dennis (2001); and b.) whether we could characterize and/or what past US monetary policy would have looked like if it was following an NGDP target (McCallum and Nelson, 1999a). The literature during this period was also in more focused on NGDP growth targets over level targets.

For a short period we see a lull in research on NGDP targets. During this period the Taylor rule, inflation targeting, and optimal policy in DSGE models became the main concern for monetary scholars. Interest in Nominal GDP targeting did not pick up again in earnest until the 2007-2008 global financial crisis. The modern NGDP targeting literature is primarily focused on questions of the uniqueness and learnability of the REE as well as the welfare implications of NGDP targeting.

One of the earliest attempts to assess determinacy and E-stability properties under NGDP targeting was Mitra (2003). Mitra analyzed the determinacy and E-stability of targeting NGDP growth precisely rather than through an interest rate rule in a simple two-equation NK model. Mitra proves that when contemporaneous growth is targeted the rational expectations equilibrium is always determinate. When agents have current endogenous variables in their information set the REE is

E-stable as well. If the central bank conducts policy assuming agents have rational expectations the REE is E-unstable for common parameterizations of the model. Mitra also finds that targeting expected growth leads to an indeterminate and E-unstable REE.

Honkapohja and Mitra (2014) study the steady-state learning properties of NGDP targeting in an NK model where interest rates are subject to the zero lower bound. They compare a Taylor rule and interest rate rules that respond to deviations from target for a price level target and an NGDP level target. All rules are vulnerable to a deflationary steady state when the interest rate hits the zero lower bound. This steady state is locally unstable under learning under both price level and NGDP level targeting. They find that price level and NGDP level targeting result in steady states with a larger domain of attraction and lower volatility of aggregate variables when forward guidance is provided by the central bank.

Jensen (2002) evaluates the desirability of NGDP targeting compared to inflation targeting in the context of a linearized model with inflation and output persistence. The policies are derived assuming the central bank has a quadratic loss function which is not necessarily the same as society's quadratic loss function. The author finds that provided there are shocks that subject society to a monetary policy trade-off, then NGDP targeting is preferred to inflation targeting.

Koenig (2012) analyzes the link between NGDP targeting and the Taylor Rule/flexible inflation targeting. The author shows that the Taylor rule is a special case of an NGDP target, but that NGDP targeting focuses more on stabilizing long-term expectations. In addition, McCallum (2015) briefly points out that NGDP growth targeting represents a special case of the optimal monetary policy

from a timeless perspective while levels tend to call for stronger responses to drops in output than the optimal policy would.

Using a medium-scale NK model, Garín et al. (2015) find that NGDP targeting leads to less welfare loss relative to a flexible price equilibrium than a Taylor rule and inflation targeting. An output gap led to only slightly less consumption loss, but only when the output gap is observed with little noise.

Sheedy (2014) demonstrates that in an overlapping generations model with debt and incomplete markets a central bank following NGDP level targeting will essentially "complete the market" by causing the value of real debt to move opposite real income. Brennan (2018a) looks at a NK model with heterogeneous preferences and financial frictions. The key result is that the consumption equivalent welfare loss under most NGDP targeting policies is less than most conventional policies. Furthermore inflation targeting becomes preferable to all of these policies when financial frictions are removed.

### 4.3 Model

The model we use for our analysis is a simple NK model, except steady state gross inflation is assumed to no longer equal one. Firms resetting their prices may partially index their future prices to trend or past inflation. The full derivation of the model is available in the appendix. The model is a slightly altered version of the one presented in Ascari and Ropele (2009).

The model when log linearized takes the following form:

$$Y_t = E_t^* Y_{t+1} - \frac{1}{\sigma_c} (i_t - E_t \pi_{t+1}) \quad (4.1)$$

$$\Delta_t = \kappa_1 Y_t + \kappa_2 s_t + \kappa_3 E_t^* \Delta_{t+1} + \kappa_4 E_t^* G_{t+1} \quad (4.2)$$

$$G_t = (1 - \varphi \beta \Pi^{(1-\theta)(\epsilon-1)}) (1 - \sigma_c) Y_t + (1 - \theta) \varphi \beta \Pi^{(1-\theta)(\epsilon-1)} E_t^* \Delta_{t+1} + \varphi \beta \Pi^{(1-\theta)(\epsilon-1)} E_t^* G_{t+1} \quad (4.3)$$

$$s_t = \xi \Delta_t + \varphi \Pi^{-\theta(\epsilon-1)} s_{t-1} \quad (4.4)$$

The endogenous variables are  $Y$ ,  $\pi$ ,  $s$ , and  $G$ . The variable  $Y$  is output. The variable  $\pi$  is inflation. The variable  $s$  is the dispersion of prices. The variable  $G$  is an auxiliary variable.  $\Delta$  is the quasi-difference of inflation equal to  $\pi_t - \omega \epsilon \pi_{t-1}$ , where  $\omega$  is the degree to which prices are indexed to trend inflation rather than past inflation, and  $\epsilon$  is the overall degree of indexation of prices.  $E_t^*$  represents possibly subjective expectations at time  $t$ .

The first equation is the NK IS curve. The second equation is a more generalized version of the Phillips curve. The third equation defines an auxiliary variable equal to the present discounted value of future marginal revenue. The fourth equation governs the evolution of price dispersion.

Here  $\sigma_c$  is the CRRA utility parameter for the consumer, the  $\kappa_i$  and  $\xi$  variables are functions of  $\theta$ , the elasticity of substitution across goods produced by firms in the economy,  $\varphi$ , which is the probability of a firm being unable to reset its price in a given period,  $\beta$ , the subjective discount factor,  $a$ , the elasticity of supply with respect to labor,  $\Pi$ , gross trend inflation,  $\epsilon$ , the degree of price indexation, and both the CRRA utility parameter as well as the disutility of labor parameter,  $\sigma_c$  and  $\sigma_n$ .



We'll focus our efforts on obtaining determinacy and E-Stability conditions for an interest rate rule which targets contemporaneous NGDP growth:

$$i_t = \alpha(\pi_t + Y_t - Y_{t-1}), \quad (4.5)$$

one that targets expected NGDP growth:

$$i_t = \alpha(E_t^* \pi_{t+1} + E_t^* Y_{t+1} - Y_t), \quad (4.6)$$

one that targets current NGDP level:

$$i_t = \alpha(p_t + Y_t), \quad (4.7)$$

and one that targets the expected level of NGDP:

$$i_t = \alpha(E_t^* p_t + E_t^* Y_t). \quad (4.8)$$

$$R_t = (P_t Y_t / \bar{P}_t Y_{ss})$$

$$\bar{P}_t = \Pi \bar{P}_{t-1}$$

$$P_t = \Pi_t P_{t-1}$$

$$\tilde{P}_t = P_t / \bar{P}_t$$

In the level equations  $p_t$  is the "price gap", the difference between the actual price level and the central bank's target price level. In addition we'll also consider the cases of these rules where the central bank has to nowcast endogenous variables.

## Stability and Determinacy

The system of questions can be written in the following form:

$$y_t = AE_t^* y_{t+1}, \quad (4.9)$$

with  $y_t = (Y_t, \pi_t, s_t, G_t, Y_{t-1}, \pi_{t-1})'$  under the contemporaneous NGDP growth rule and  $Y_{t-1}$  is dropped for the expected NGDP growth and both NGDP level rules, but  $p_t$  is added for the level rules. The condition for determinacy is that the number of eigenvalues of  $A$  less than one in absolute value must equal to the number of "free" variables, those which depend upon expectations of future values of themselves. The free variables under both rules are  $Y$ ,  $\pi$ , and  $G$ . If the number of eigenvalues inside the unit circle is exactly three then the steady state is determinate, less than three then the steady state will be indeterminate. If the number of eigenvalues inside the unit circle is more than three that will imply that the steady state is explosive.

## E-Stability

We can rewrite the system of equations in a slightly different form:

$$y_t = ME_t^* y_{t+1} + Ny_{t-1} \quad (4.10)$$

where  $y = (Y, \pi, s, G)'$ . Under least squares learning agents collect data on the endogenous variables in the economy and try to estimate the equilibrium relationship between them. This provides a more believable explanation of how agents in the model form expectations. At time  $t$ , the agent believes that the relationship between endogenous variables is:

$$y_t = a_{t-1} + b_{t-1}y_{t-1}. \quad (4.11)$$

Here  $a_t$  and  $b_t$  are the agent's estimates of structural relationships of the economy. Using this perceived law of motion (PLM) agents form expectations of future variables:

$$E_t^* y_{t+1} = a_{t-1} + b_{t-1}y_t. \quad (4.12)$$

We can substitute these expectations into (4.10) to obtain the actual law of motion (ALM).

$$y_t = (I - Mb_t)^{-1}Ma_t + (I - Mb_t)^{-1}Ny_{t-1} \quad (4.13)$$

The coefficient matrices in the PLM are estimated using recursive least squares. The algorithm used is:

$$\begin{aligned} \zeta_t &= \zeta_{t-1} + t^{-1}R_t^{-1}e_{t-1}\zeta_t' \\ R_t &= R_{t-1} + t^{-1}(e_{t-1}e_{t-1}' - R_{t-1}), \end{aligned}$$

where  $\zeta_t = (a_t, b_t)'$ ,  $e_t' = (1, y_t')$ , and  $\varsigma_t = y_t - \zeta_{t-1}'e_{t-1}$ . We are concerned with whether expectations will converge to the rational expectations equilibrium (REE) which is in the form of the MSV solution:

$$y_t = a + by_{t-1}. \quad (4.14)$$

We can map the coefficients in the ALM to the estimates being used in the PLM.

$$T(a, b) = ((I - Mb)^{-1}Ma, (I - Mb)^{-1}N) \quad (4.15)$$

By the E-stability principal, the algorithm will converge to the REE if the REE is E-stable. E-stability is governed by the differential equation:

$$\frac{d}{d\tau}(a, b, c) = T(a, b, c) - (a, b, c). \quad (4.16)$$

Following Evans and Honkapohja (2001) in models of the form expressed above E-stability depends upon the eigenvalues of the following:

$$DT_a(\bar{a}, \bar{b}) = (I - M\bar{b})^{-1}M \quad (4.17)$$

$$DT_b(\bar{b}) = [(I - M\bar{b})^{-1}N]' \otimes [(I - M\bar{b})^{-1}M] \quad (4.18)$$

With information set  $(y'_t, 1)$  a necessary condition for E-stability is that the eigenvalues of  $DT_a$  and  $DT_b$  have real parts less than 1. We choose to examine this information set even though agents having knowledge of contemporaneous endogenous variables is unrealistic because it can be seen as possibly one or two steps down from rational expectations. Whereas when agents only know  $y_{t-1}$  we step further away.

The E-stability condition changes when we assume that agents nowcast endogenous variables. If that is the case then the system becomes:

$$y_t = PE_t^*y_t + ME_t^*y_{t+1} + Ny_{t-1} \quad (4.19)$$

The agents still forecast  $y_t$  using (4.14). This means that the nowcast of  $y_t$  is equal to:

$$E_t^* y_t = a_{t-1} + b_{t-1} y_{t-1}. \quad (4.20)$$

Their forecast of  $y_{t+1}$  is now:

$$E_t^* y_{t+1} = (1 + b_{t-1}) a_{t-1} + b_{t-1}^2 y_{t-1}. \quad (4.21)$$

These lead to an ALM of:

$$y_t = (P + M(1 + b_{t-1})) a_{t-1} + (N + P b_{t-1} + b_{t-1}^2) y_{t-1}. \quad (4.22)$$

The map between the PLM and ALM is:

$$T(a, b) = ((P + M(1 + b))a, Mb^2 + Pb + N). \quad (4.23)$$

For E-stability of the REE we need the eigenvalues of the following matrices evaluated at the REE to have real parts less than one:

$$DT_a(\bar{a}, \bar{b}) = P + M(1 + \bar{b}) \quad (4.24)$$

$$DT_b(\bar{b}) = \bar{b}' \otimes M + I \otimes M\bar{b} + I \otimes P \quad (4.25)$$

## Calibration

We calibrate our model to be in line with those presented in Galí (2015, Chapter 3). In our results section we will present four graphs for each interest rate rule which show the determinacy, E-stability, and covariance stationarity properties

TABLE 18. Model Calibration

Parameter	Value
$a$	0.75
$\beta$	0.99
$\theta$	9
$\varphi$	0.75
$\sigma_n$	5
$\epsilon$	$\{0, 0.5\}$
$\omega$	$\{0, 0.5\}$
$\sigma_c$	$\{0.157, 1, 6.1\}$

for different combinations of the interest rate response parameter and the value of gross trend inflation for different parameterizations of  $\sigma_c$ ,  $\epsilon$ , and  $\omega$ . Each rule will have the values of  $\theta$ ,  $\beta$ ,  $\varphi$ ,  $a$ , and  $\sigma_n$  in common across all graphs. The values of these parameters are displayed in Table 1. We consider three values of  $\sigma_c$  which correspond to values used in Woodford (1999), Clarida et al. (1999), and McCallum and Nelson (1999b).

The reason we do not consider full indexation is because the model collapses to the simple NK model with zero trend inflation.

#### 4.4 Results

Our main question is for what combinations of  $\Pi$  and  $\alpha$  do we see determinacy and E-stability. We present our results for four different parameterizations for each rule. We vary the CRRA utility parameter and the degree of indexation to trend inflation to see what their effects are.

The values of  $\alpha$  under consideration are from zero to seven. Larger ranges were explored but did not change the results.

The values of  $\Pi$  under consideration depend on the value of indexation. As can be seen in the appendix, there are values of  $\Pi$  for which there is no solution to

the firm's pricing problem because the expected discounted sums of future marginal revenues and marginal costs will not converge. When  $\epsilon$  is equal to one half that upper bound on  $\Pi$  is about 1.068, and when  $\epsilon$  is equal to zero that upper bound on  $\Pi$  is closer to 1.033. These are quarterly rates, in our figures we will display the annualized trend inflation rates which are 14% and 30% respectively.

We provide four figures for each rule. Panels a through c will always involve the indexation parameter,  $\epsilon$  equal to 0.5, while in panel d it will be set to 0. In panels a and d we set  $\sigma_c = 6.1$ , in panel b we set  $\sigma_c = 1$ , and in panel c we set it equal to 0.157.

The results for the contemporaneous NGDP growth rules are presented in figures 1 and 2. Common to all of these figures is a range of values for trend inflation for which any  $\alpha > 1$  will result in both determinacy and E-stability. For panels a through c, when there is partial indexation, the upper limit on trend inflation for this region appears to be around 21% annually and for panel d where there is no indexation it appears to occur a little above 10%. At values of trend inflation below those thresholds, determinacy and E-stability being ensured with  $\alpha > 1$  is in line with the results found in Brennan (2018b) for the zero trend inflation steady state. Trend inflation above those limits lead to a loss of determinacy and E-stability although not necessarily both are lost immediately. The introduction of nowcasting mostly led to minor shifts in what parameter values were E-stable or not.

The key takeaway here is that even up to fairly large values of annual trend inflation a value of  $\alpha$  greater than one will ensure determinacy and E-stability. This result still holds when we assume a more realistic information set for the central bank and require it to nowcast current inflation and output.

The expected growth rule with full information is in figure 3. It appears to inherit some of the determinacy and E-Stability properties it had under zero trend inflation for values of trend inflation below the previously discussed limit. While the general bounds appear roughly the same, either the upper or lower bound on values of  $\alpha$  which ensure determinacy and E-stability now change with the level of trend inflation. Higher rates of trend inflation in fact lead to a wider range of values of  $\alpha$  which guarantee determinacy and E-stability. Once the upper limit is reached there appears to be a flipping in the range of values of  $\alpha$  which lead to determinacy and E-stability. The nowcasting rule is presented in figure 4 does not lead to many large changes beyond there being virtually no combinations of  $\alpha$  and  $\Pi$  that yield a determinate and E-stable equilibrium.

One very striking result here is that while in previous work setting  $\sigma_c$  equal to one led to indeterminacy and E-instability for all values of  $\alpha$ , when trend inflation goes above 21% annually it appears that the REE is determinate and E-stable for all values of  $\alpha$  up to a certain level of trend inflation. Interestingly under the nowcasting rule in that same range of values for  $\Pi$ , the values of  $\alpha$  greater than one lead to an E-unstable REE.

The contemporaneous nominal GDP level targets are presented in figures 5 and 6. Just as with previous figures up to a very high upper bound on trend inflation the results for this interest rate rule found in previous work. Additionally the nowcasting version of the interest rate rule does not change the E-stability results much.

Finally in figure 7 we display results for the interest rate rule which targets the expected level of NGDP. For all values of trend inflation under a certain limit, there is a range of values for  $\alpha$  which will produce both determinacy and



E-stability. This range starts at zero and has an upper limit that is increasing with  $\sigma_c$ . This is the same result that was found in the zero trend inflation case.

Across all figures some overall trends appear. First the indexation parameter only appears to affect at what rate of trend inflation for which the firm's optimization problem has no solution. Second higher levels of trend inflation are associated with the emergence of indeterminate and E-stable regions, implying the possibility of learnable sunspots.

## 4.5 Conclusion

In summary, we have laid out the conditions for determinacy and E-stability of the REE in a NK model with positive trend inflation and partial indexation of prices under a variety of interest rate rules which target either the level or growth of NGDP. Our results build on previous research in the areas of non-zero trend inflation in NK models and the desirability of NGDP targets. For all interest rate rules the determinacy and E-stability properties of the zero trend inflation case mostly carry over up until some higher limit for trend inflation which is increasing with the degree of inflation indexation among firms. We find that as was the case under zero trend inflation the NGDP level target is determinate and E-stable for all values of the interest rate response parameter,  $\alpha$ , that were considered and have no reason to believe that would change. There is now, however, the risk of explosive solutions under all of the interest rate rules. Interest rate rules which targeted the expected level or growth rate of NGDP fared poorly depending upon the value of  $\sigma_c$ . This is particularly concerning when the central bank has to nowcast current levels of real output for the expected NGDP growth interest rate rule.

Before seriously considering NGDP targets as a policy alternative there are different future avenues of research that should be pursued. One of these is whether

### Determinacy and E-Stability Under NGDP Growth Rule

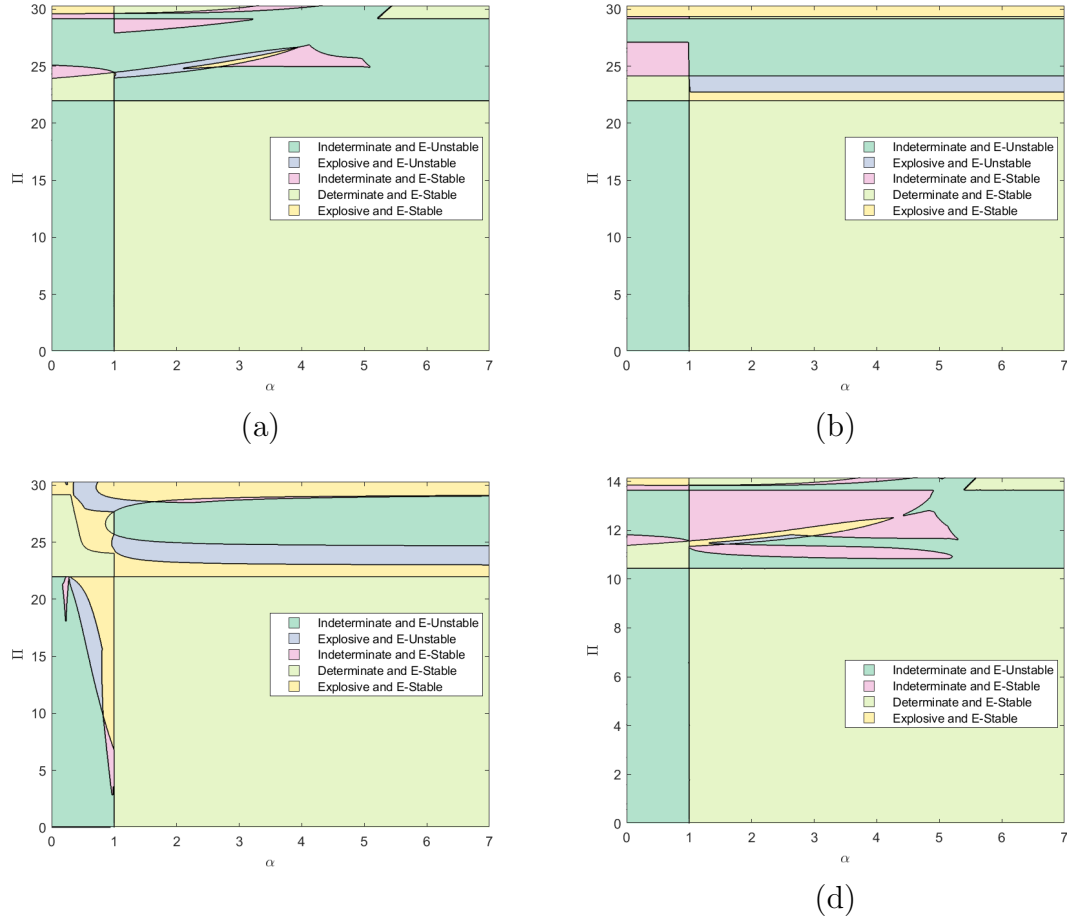


FIGURE 17. In the above figures, the information set is assumed to include contemporaneous endogenous variables. In figures, a through c price indexation is set to 0.5 while in figure d it is set to 0. In figures a and d the CRRA utility parameter is set to 6.1, in figure b it is set to 1, and in figure c it is set to 0.157.

## Determinacy and E-Stability Under Nowcasting NGDP Growth Rule

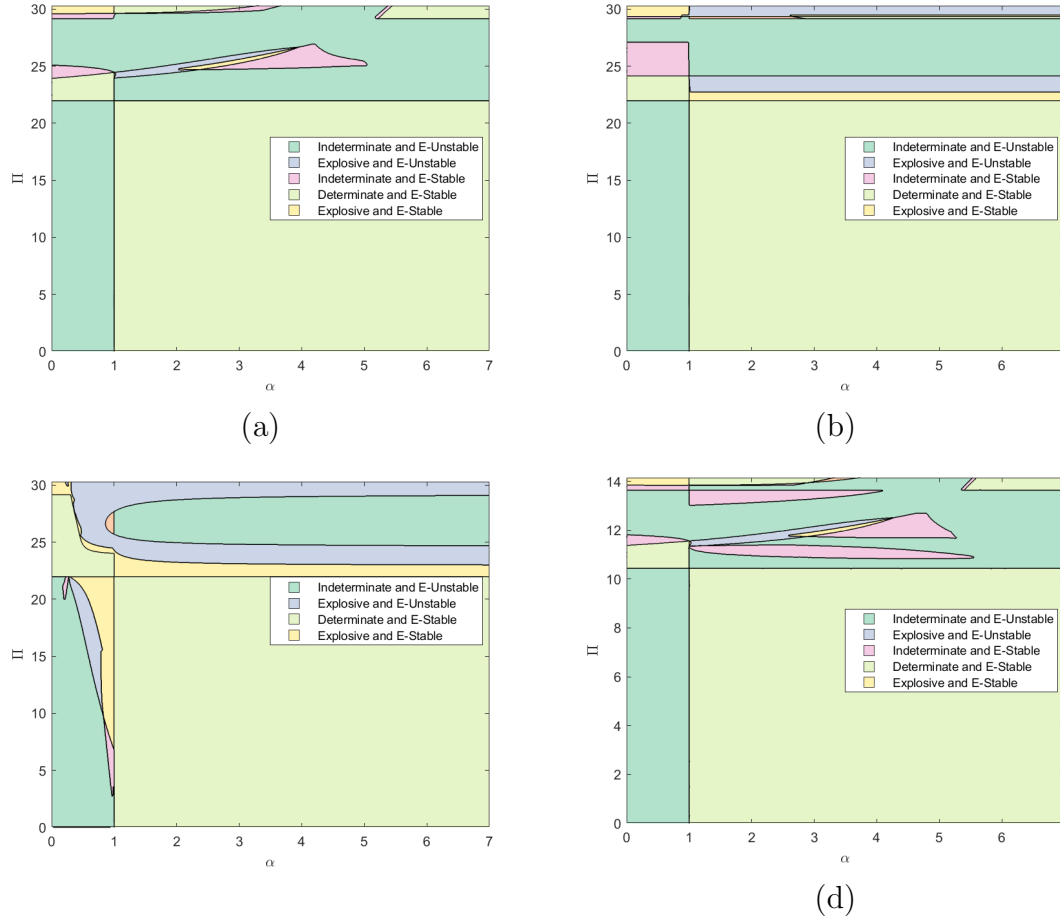


FIGURE 18. In the above figures the information set is assumed to not include contemporaneous endogenous variables. In figures a through c price indexation is set to 0.5 while in figure d it is set to 0. In figures a and d the CRRA utility parameter is set to 6.1, in figure b it is set to 1, and in figure c it is set to 0.157.

## Determinacy and E-Stability Under Expected NGDP Growth Rule

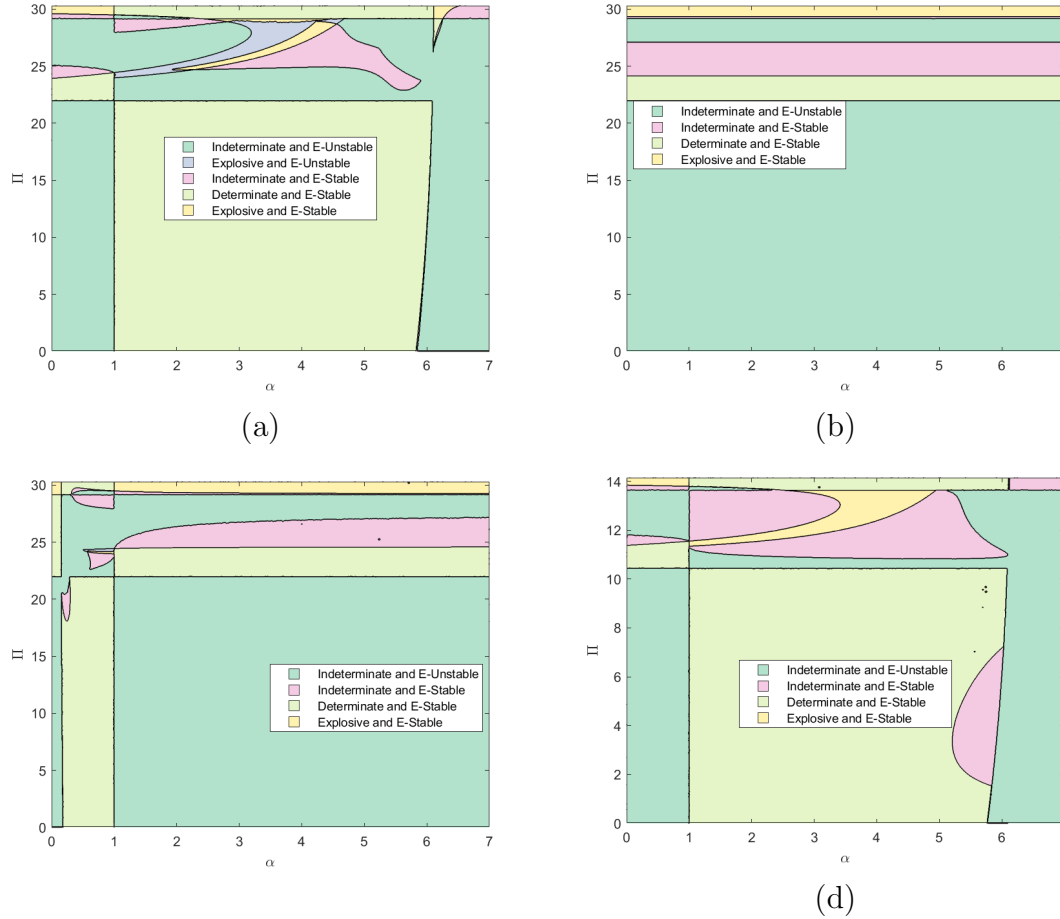


FIGURE 19. In the above figures, the information set is assumed to include contemporaneous endogenous variables. In figures a through c price indexation is set to 0.5 while in figure d it is set to 0. In figures a and d the CRRA utility parameter is set to 6.1, in figure b it is set to 1, and in figure c it is set to 0.157.

## Determinacy and E-Stability Under Nowcasting Expected NGDP Growth Rule

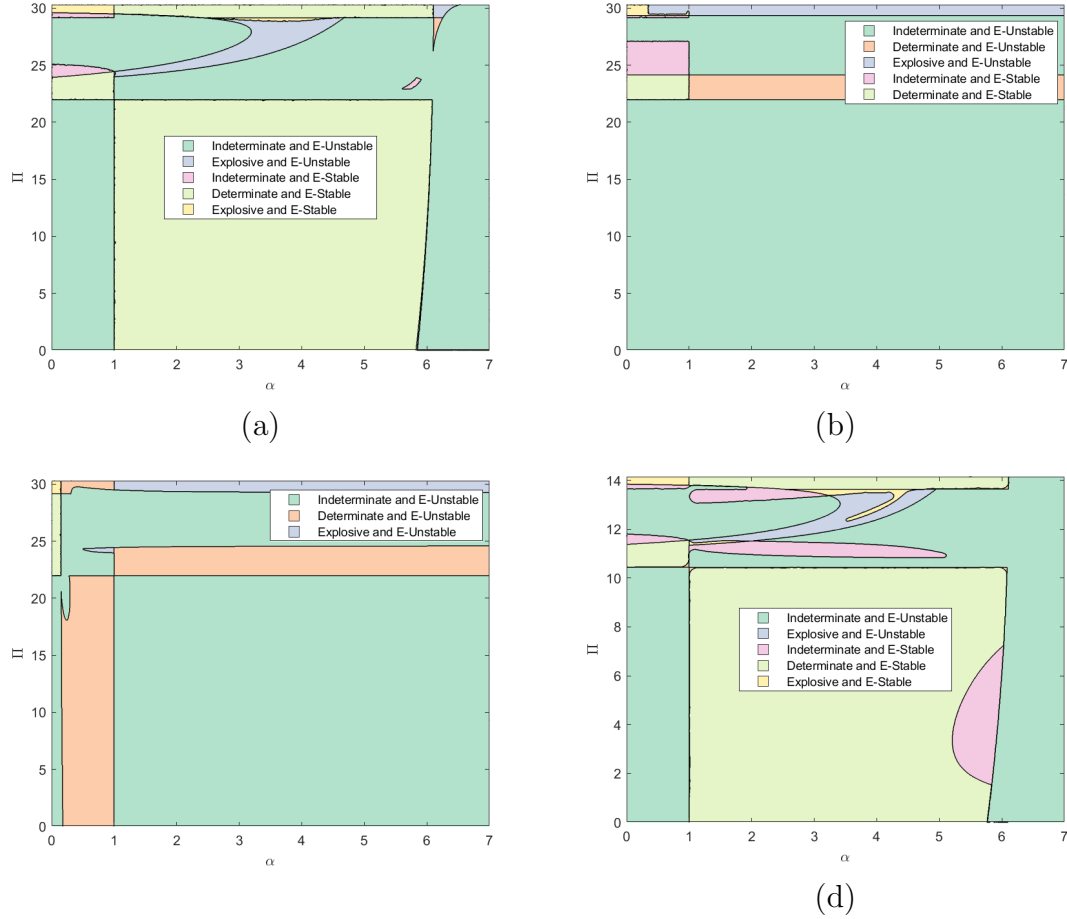


FIGURE 20. In the above figures the information set is assumed to not include contemporaneous endogenous variables. In figures a through c price indexation is set to 0.5 while in figure d it is set to 0. In figures a and d the CRRA utility parameter is set to 6.1, in figure b it is set to 1, and in figure c it is set to 0.157.

### Determinacy and E-Stability Under NGDP Level Rule

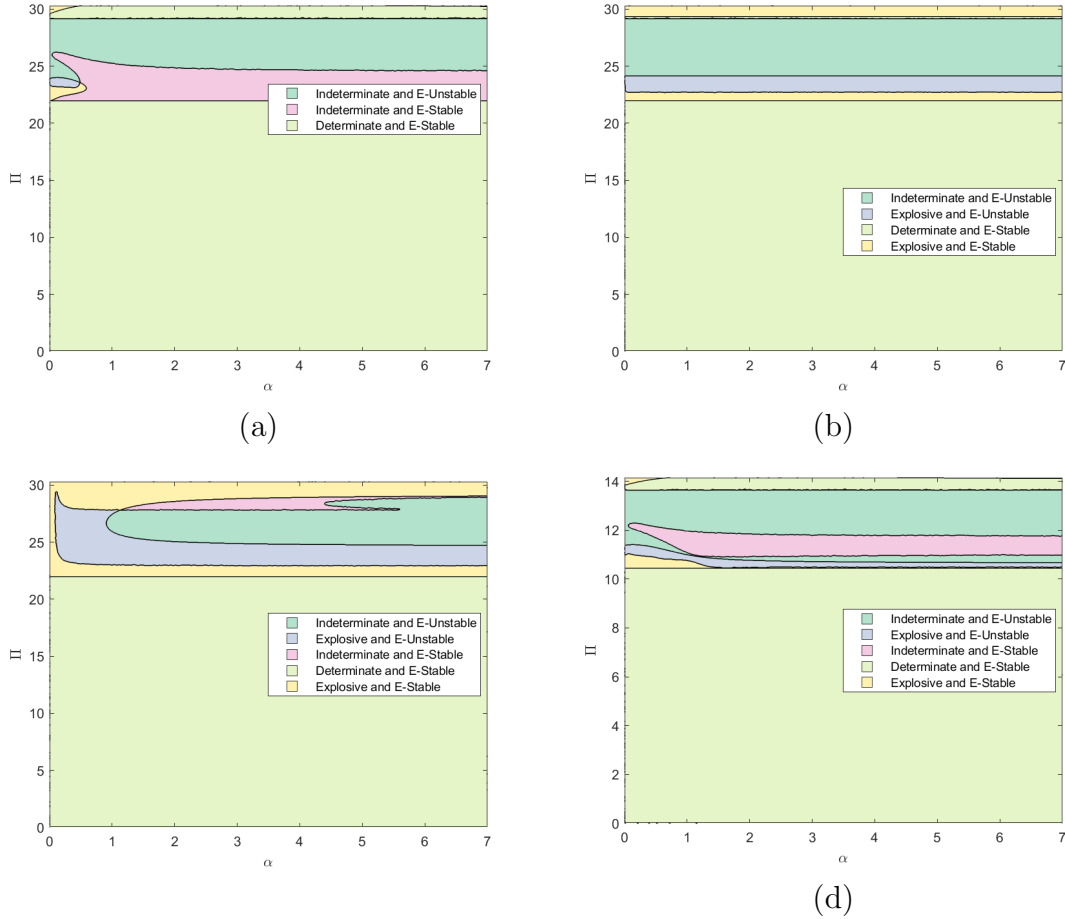


FIGURE 21. In the above figures, the information set is assumed to include contemporaneous endogenous variables. In figures a through c price indexation is set to 0.5 while in figure d it is set to 0. In figures a and d the CRRA utility parameter is set to 6.1, in figure b it is set to 1, and in figure c it is set to 0.157.

## Determinacy and E-Stability Under Nowcasting NGDP Level Rule

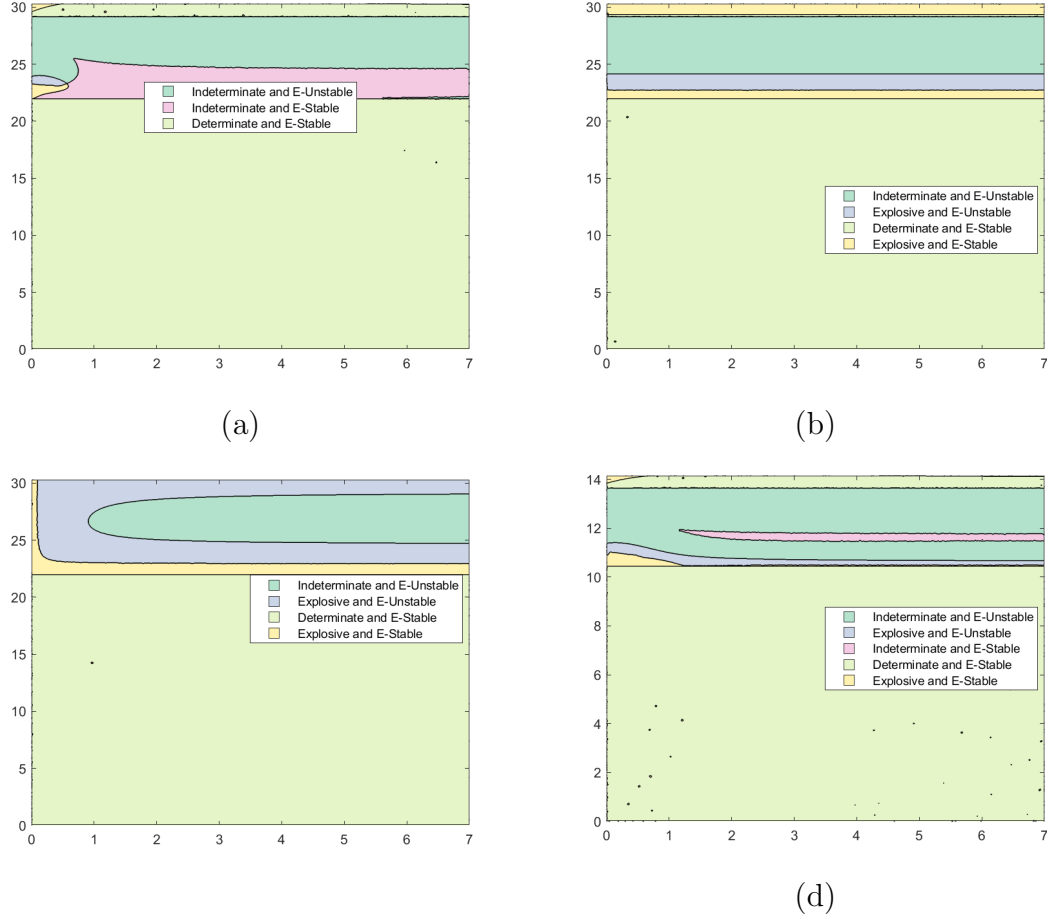


FIGURE 22. In the above figures the information set is assumed to not include contemporaneous endogenous variables. In figures a through c price indexation is set to 0.5 while in figure d it is set to 0. In figures a and d the CRRA utility parameter is set to 6.1, in figure b it is set to 1, and in figure c it is set to 0.157.

### Determinacy and E-Stability Under Expected NGDP Level Rule

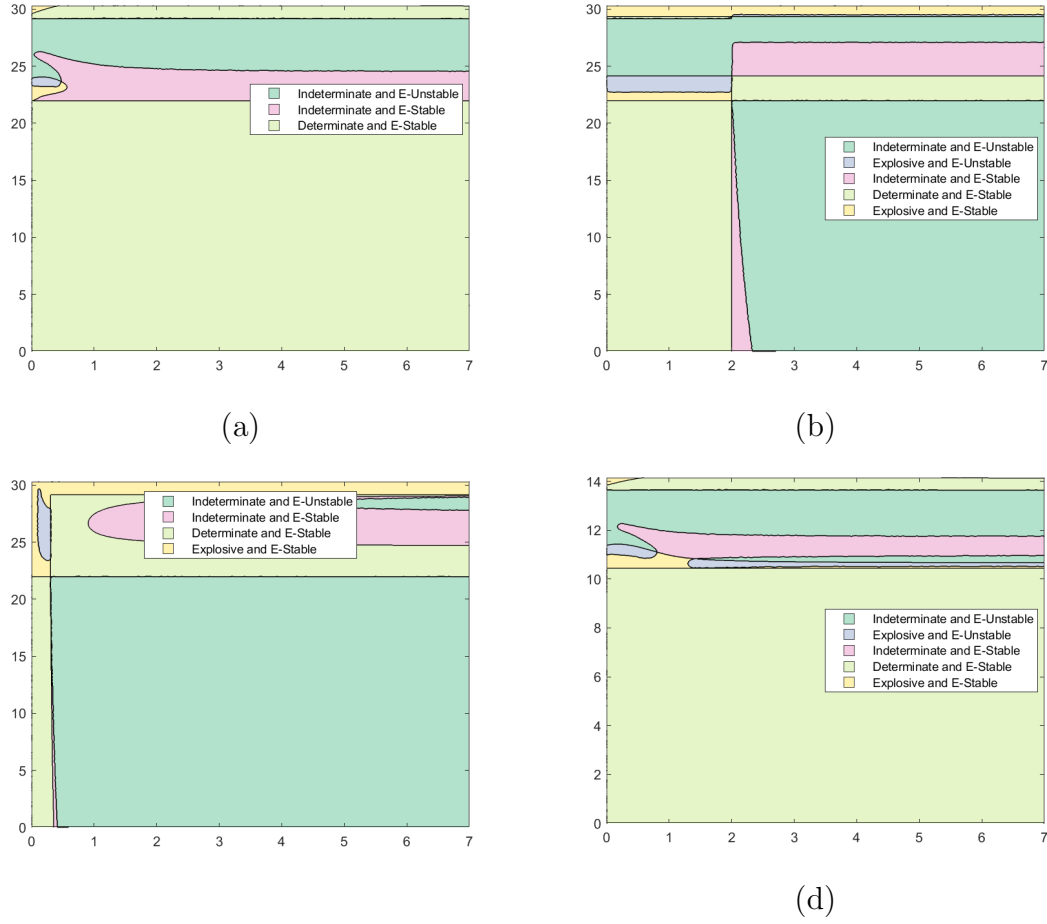


FIGURE 23. In figures a through c price indexation is set to 0.5 while in figure d it is set to 0. In figures a and d the CRRA utility parameter is set to 6.1, in figure b it is set to 1, and in figure c it is set to 0.157.



these results change if changes to interest rates are implemented with a lag. We also may want to turn our eye towards distributional concerns of trend inflation and NGDP targets. An important question to ask with regard to monetary models with heterogeneous agents is who bears the costs of trend inflation in terms of lost consumption and how the agents benefit or lose under NGDP targeting as compared to conventional monetary policy such as a Taylor rule.

## APPENDIX A

### SUPPLEMENTAL MATERIAL FOR CHAPTER II

In this appendix we describe the derivation of the model presented in Section 3 of the paper. There is a continuum of households  $i \in [0, 1]$ . Household  $i$  seeks to maximize the discounted expected present value of utility which is a function of real consumption, real money balances, and labor supply.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{1}{1-\gamma} \left( \frac{M_t}{P_t} \right)^{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right] \quad (\text{A.0.1})$$

Where  $N_t$  is hours worked,  $M_t/P_t$  is real money balances with  $P_t$  defined as:

$$P_t \equiv \left[ \int_0^1 p_{jt}^{1/(\mu_t-1)} dj \right]^{\mu_t-1} \quad (\text{A.0.2})$$

$C_t$  is a composite consumption good aggregated from the differentiated goods that are produced by the  $j \in [0, 1]$  monopolistically competitive firms. In terms of the goods produced by these firms  $C_t$  is:

$$C_t \equiv \left[ \int_0^1 c_{jt}^{\frac{1}{\mu_t}} dj \right]^{\mu_t}. \quad (\text{A.0.3})$$

Where  $\mu_t = \mu + \xi_t$  is the markup over marginal cost that firms set prices at when prices are flexible, with  $\mu > 1$  is the mean markup level and  $\xi_t$  is a covariance stationary AR-1 process with an unconditional mean of zero. For any level of  $C_t$  the household chooses they must choose the mixture of  $c_{jt}$  that minimizes expenditures.

$$\begin{aligned} \min_{c_{jt}} & \int_0^1 p_{jt} c_{jt} dj \\ \text{s.t.} & \left[ \int_0^1 c_{jt}^{\frac{1}{\mu_t}} dj \right]^{\mu_t} \geq C_t \end{aligned}$$

The first order condition is:

$$p_{jt} - \Lambda_t \left[ \int_0^1 c_{jt}^{\frac{1}{\mu_t}} dj \right]^{\mu_t-1} c_{jt}^{\frac{1-\mu_t}{\mu_t}} = 0. \quad (\text{A.0.4})$$

Here  $\Lambda_t$  is the Lagrangian multiplier. The first order condition can be rearranged to  $c_{jt} = (p_{jt}/\Lambda_t)^{-\theta} C_t$ . Substituting this into (A.0.3) and solving for  $\Lambda_t$  it can be shown that  $\Lambda_t = P_t$ , so we can conclude that the demand for the good made by firm  $j$  is:

$$c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\frac{\mu_t}{\mu_t-1}} C_t. \quad (\text{A.0.5})$$

The real household budget constraint is:

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \Pi_t. \quad (\text{A.0.6})$$

Where  $B_t$  is nominal one period bond holdings,  $i_t$  is the nominal interest rate paid on bonds,  $W_t$  is the nominal wage earned by the household, and  $\Pi_t$  are the real profits received from firms. In addition to the above flow budget constraint households also follow a no ponzi-game condition:

$$\lim_{T \rightarrow \infty} \beta^{T-t} E_t \frac{C_T^{-\sigma}}{C_t^{-\sigma}} \frac{B_T}{P_T} \geq 0. \quad (\text{A.0.7})$$

The first order conditions for the household's optimization problem are:

$$C_t^{-\sigma} = \beta(1 + i_t)E_t\left(\frac{P_t}{P_{t+1}}\right) C_{t+1}^{-\sigma} \quad (\text{A.0.8})$$

$$\frac{\left(\frac{M_t}{P_t}\right)^{-\gamma}}{C_t^{-\sigma}} = \frac{i_t}{1 + i_t} \quad (\text{A.0.9})$$

$$\frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \quad (\text{A.0.10})$$

The first equation is the household's Euler equation of intertemporal choice.

The second equation determines the household's money demand, it says that the marginal rate of substitution between real money holdings and consumption will equal the opportunity cost of holding money. The third equation determines household labor supply, it says that the marginal rate of substitution between leisure and consumption will equal the real wage.

Firms are monopolistically competitive and sell differentiated products. Firm  $j$  has the following production function:

$$Y_{jt} = Z_t N_{jt}^a \quad (\text{A.0.11})$$

Here  $Z_t$  is an aggregate productivity disturbance,  $E(Z_t) = 1$  and  $\log Z_t = \rho \log Z_{t-1} + \epsilon_t$  where  $\epsilon$  is an i.i.d. mean zero shock. The firm's real marginal costs can be written as:

$$\psi_{jt} = \frac{W_t}{P_t Z_t N_{jt}^{a-1}} = \frac{W_t}{a P_t Y_{jt} / N_{jt}}. \quad (\text{A.0.12})$$

Firms are not always able to adjust their price each period. As in Calvo (1983) only a fraction  $1 - \omega$  are able to update their prices, the rest of the firms do not update their prices. When the firm is making this decision it does so choosing

the price that maximizes the discounted stream of future real profits if it is unable to change its price, therefore its pricing decision problem can be written as:

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right) Y_{jt+i} - \psi_{jt+i} Y_{jt+i} \right] \quad (\text{A.0.13})$$

Where  $\Delta_{t+i}$  is equal to  $\beta^i (C_{t+i}/C_t)^{-\sigma}$ . We can use the demand equation for the good firm  $j$  produces to simplify this.

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\frac{1}{\mu_{t+i}}} - \psi_{jt+i} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\frac{\mu_t}{\mu_{t+i}-1}} \right] C_{t+i} \quad (\text{A.0.14})$$

The first order condition for the optimal price  $p_t^*$  with some manipulation is:

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,t+i} \frac{1}{\mu_{t+i} - 1} C_{t+i} \left( \frac{p_t^*}{P_{t+i}} \right)^{\frac{-\mu_{t+i}}{\mu_{t+i}-1}} \frac{1}{P_{t+i}} = E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,t+i} C_{t+i} \frac{-\mu_{t+i}}{\mu_{t+i} - 1} \psi_{jt+i} \left( \frac{1}{p_t^*} \right) \left( \frac{p_t^*}{P_{t+i}} \right)^{\frac{-\mu_{t+i}}{\mu_{t+i}-1}}. \quad (\text{A.0.15})$$

We can use the first order condition to find the flexible price output level, or potential GDP. Setting  $\omega = 0$  the optimal price becomes:

$$\frac{p_t^*}{P_t} = \mu_t \psi_{jt}. \quad (\text{A.0.16})$$

The price the firm sets is just equal to markup times the nominal marginal cost. Furthermore since all firms face the same problem  $p_t^* = P_t$ , which implies that  $\psi_{jt} = 1/\mu_t$ . Using the definition of real marginal costs from above we have that  $W_t/P_t = aY_{jt}/\mu N_{jt}$ . Using this with the household's intratemporal optimality condition between leisure and consumption we get that:

$$\frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{a Y_t}{\mu_t N_t}. \quad (\text{A.0.17})$$

Because  $C_t = Y_t$  in equilibrium and solving for  $N_t = (Y_t/Z_t)^{1/a}$  in the production function we can obtain the flexible price level of output with the above equation.

$$Y_t^f = \left( \frac{a}{\chi \mu_t} \right)^{\frac{a}{1+a(\sigma-1)+\eta}} Z_t^{\frac{1+\eta}{1+a(\sigma-1)+\eta}} \quad (\text{A.0.18})$$

From here we can define the "natural" rate of output as the one where mark-ups are equal to their mean value, so  $Y_t^n$  is the same as above but with  $\mu$  in place of  $\mu_t$ .

Going back to the definition of  $P_t$  when prices are sticky it must be the case that  $(1 - \omega)$  of the  $p_{jt}$  are equal to  $p^*$ . For those that can't update their price because they were chosen randomly the average of their prices must be  $P_{t-1}$ . Therefore the price level satisfies:

$$P_t^{\frac{1}{\mu_t-1}} = (1 - \omega)(p_t^*)^{\frac{1}{\mu_t-1}} + \omega P_{t-1}^{\frac{1}{\mu_t-1}}. \quad (\text{A.0.19})$$

The equilibrium is a collection  $\{C_t, N_t, M_t, P_t, \psi_t, p_t^*, W_t, i_t\}_0^\infty$  such that (A.0.8)-(A.0.12), (A.0.15), (A.0.17) and (A.0.19) all hold.

To derive the New Keynesian Phillips Curve used in this paper, start with equations (A.0.15) and (A.0.19), which represent the optimal pricing decision of a firm and the evolution of the price level respectively. Define  $Q_t \equiv p_t^*/P_t$ . In steady state  $Q_t = Q_{t+1} = 1$ . If we divide the equation for the evolution of the price level by  $P_t$  it becomes:

$$1 = (1 - \omega)Q_t^{\frac{1}{\mu_t-1}} + \omega \left( \frac{P_{t-1}}{P_t} \right)^{\frac{1}{\mu_t-1}}. \quad (\text{A.0.20})$$

Assuming that steady state inflation is zero  $Q_t^{\frac{1}{\mu_t-1}} \approx (1 + \frac{1}{\mu-1}q_t)$ ,  $(P_{t-1}/P_t)^{\frac{1}{\mu_t-1}} \approx (1 - \frac{1}{\mu-1}\pi_t)$  where  $q_t$  and  $\pi_t$  are percentage deviations from steady state, we can rewrite the equation that governs the evolution of the price level as a linear relationship:

$$q_t = \frac{\omega}{1 - \omega} \pi_t. \quad (\text{A.0.21})$$

Next we return to the first order condition for the firm's optimization problem and factor out the mean value of  $\mu_{t+i}$ :

$$\begin{aligned} \mu E_t \sum_{i=0}^{\infty} \omega^i \beta^i \psi_{jt+i} C_{t+i}^{1-\sigma} (1 + \frac{1}{\mu} \xi_{t+i}) (1 + \frac{1}{\mu-1} \xi_{t+i}) \frac{1}{p_t^*} \left( \frac{p_t^*}{P_{t+i}} \right)^{\frac{-\mu_{t+i}}{\mu_{t+i}-1}} = \\ E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} (1 + \frac{1}{\mu-1} \xi_{t+i}) \frac{1}{P_{t+i}} \left( \frac{p_t^*}{P_{t+i}} \right)^{\frac{-\mu_{t+i}}{\mu_{t+i}-1}}. \end{aligned}$$

We will use the following approximations of each variable in terms of percentage deviations from steady state values,  $C_{t+i}^{1-\sigma} \approx C^{1-\sigma}(1 + (1 - \sigma)c_{t+i})$ ,  $(p_t^*/P_{t+i})^{-\mu_{t+i}/(\mu_{t+i}-1)} \approx (1 + \mu(\mu-1)^{-1}(p_{t+i} - \hat{p}_t^*))$ , and  $\psi_{jt+i} \approx \psi(1 + \hat{\psi}_{jt+i})$  where  $C$  and  $\psi$  are the steady state values of consumption and real marginal costs.

Plugging these approximations into both sides, exploiting the fact that  $\mu\psi = 1$ , and canceling out like terms obtains the following:

$$\begin{aligned}
E_t \sum_{i=0}^{\infty} \omega^i \beta^i (\hat{\psi}_{jt+i} - \frac{1}{\mu(\mu-1)} \xi_{t+i} + \frac{\mu}{\mu-1} (p_{t+i} - \hat{p}_t^*) - \hat{p}_t^*) = \\
E_t \sum_{i=0}^{\infty} \omega^i \beta^i (\frac{\mu}{\mu-1} (p_{t+i} - \hat{p}_t^*) - p_{t+i} - \frac{1}{\mu-1} \xi_{t+i}).
\end{aligned} \tag{A.0.22}$$

To proceed we will need to rewrite  $\hat{\psi}_{jt+i}$  in terms of average marginal costs  $\psi_t = (W_t/P_t)/(aC_t/N_t)$  where we define  $N_t$  as the average labor supply.

$$N_t = \int_0^1 N_{jt} dj = \int_0^1 \left( \frac{Y_{jt}}{Z_t} \right)^{\frac{1}{a}} dj = \left( \frac{Y_t}{Z_t} \right)^{\frac{1}{a}} \int_0^1 \left( \frac{p_{jt}}{P_t} \right)^{\frac{-\mu_t}{a(\mu_t-1)}} dj \tag{A.0.23}$$

When linearized the last term drops out because it becomes  $1 - (\mu/a(\mu - 1)) \int (\hat{p}_{jt} - p_t) dj$  and when (A.0.2) is linearized we get that  $p_t = \int \hat{p}_{jt} dj$ . So as a result:

$$\hat{\psi}_t = (w_t - p_t) - (y_t - n_t) = (w_t - p_t) - \frac{a-1}{a} y_t - \frac{1}{a} z_t. \tag{A.0.24}$$

And using the equation for  $\psi_{jt}$  we can get its linear approximation as:

$$\hat{\psi}_{jt} = (w_t - p_t) - \frac{a-1}{a} y_{jt} - \frac{1}{a} z_t. \tag{A.0.25}$$

The difference between the two is:

$$\hat{\psi}_{jt} - \hat{\psi}_t = \frac{1-a}{a} (y_{jt} - y_t). \tag{A.0.26}$$

Finally we can employ the demand relationship all firms face to write this in terms of prices.



$$\hat{\psi}_{jt} = \hat{\psi}_t - \frac{\mu(a-1)}{a(\mu-1)}(\hat{p}_{jt} - p_t) \quad (\text{A.0.27})$$

Plugging the above relationship into equation (A.0.22), using the fact that  $\hat{p}_{jt} = \hat{p}_t^*$ , and after some manipulation we get:

$$\hat{p}_t^* = \frac{a(\mu-1)(1-\omega\beta)}{a-\mu} \sum_{i=0}^{\infty} \beta^i \omega^i \left( \frac{a-\mu}{a(\mu-1)} p_{t+i} - \hat{\psi}_{t+i} + \frac{1-\mu}{\mu(\mu-1)} \xi_{t+i} \right). \quad (\text{A.0.28})$$

To simplify further recognize that if we pull out the current period variables from the summation the summation is now equal to  $\omega\beta E_t \hat{p}_{t+1}^*$ :

$$\hat{p}_t^* = (1-\omega\beta)p_t + \frac{a(\mu-1)(1-\omega\beta)}{\mu-a} \psi_t + \frac{a(1-\mu)(1-\omega\beta)}{\mu(a-\mu)} \xi_t + \omega\beta E_t \hat{p}_{t+1}^*. \quad (\text{A.0.29})$$

We can substitute for  $\hat{p}_t^* = q_t + p_t$  and use equation (A.0.21) to get:

$$\begin{aligned} \frac{\omega}{1-\omega} \pi_t &= \frac{a(\mu-1)(1-\omega\beta)}{\mu-a} \psi_t + \frac{a(1-\mu)(1-\omega\beta)}{\mu(a-\mu)} \xi_t + \frac{\omega\beta}{1-\omega} E_t \pi_{t+1} \\ \pi_t &= \tilde{\lambda} \hat{\psi}_t + \frac{\tilde{\lambda}}{\mu} \xi_t + \beta E_t \pi_{t+1}. \end{aligned}$$

where

$$\tilde{\lambda} = \frac{(\mu-1)(1-\omega)(1-\omega\beta)a}{\omega(\mu-a)}$$

Recall that  $\psi_t = (W_t/P_t)/(aY_t/N_t)$ , so  $\hat{\psi}_t = w_t - p_t - (y_t - n_t)$ . The linearized form of equation (A.0.10) yields  $w_t - p_t = \eta n_t + \sigma y_t$ . Finally by the production function  $n_t = (1/a)(y_t - z_t)$ . Using these we get:

$$\begin{aligned}
\hat{\psi}_t &= \frac{\eta}{a}(y_t - z) + \sigma y_t - (y_t - \frac{1}{a}(y_t - z_t)) \\
&= \frac{\sigma a + \eta - a + 1}{a} y_t - \frac{1 + \eta}{a} z_t \\
&= \frac{\sigma a + \eta - a + 1}{a} \left[ y_t - \frac{1 + \eta}{\sigma a + \eta - a + 1} z_t \right].
\end{aligned}$$

Now note that the linearization of the natural level of output is equal the second term inside the square brackets.

$$\hat{\psi}_t = \frac{\sigma a + \eta - a + 1}{a} (y_t - y_t^n) \quad (\text{A.0.30})$$

So now the New Keynesian Phillips Curve assumes the form used in the main body of the paper:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t \quad (\text{A.0.31})$$

$$\lambda = \frac{\sigma a + \eta - a + 1}{a} \tilde{\lambda}. \quad (\text{A.0.32})$$

where  $x_t \equiv y_t - y_t^n$ , and  $u_t = \tilde{\lambda} \xi_t / \mu$ .

The next step of deriving the linearized model is to obtain the New Keynesian IS equation. We start by linearizing the household Euler equation:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}). \quad (\text{A.0.33})$$

If we subtract  $y_t^n$  and  $E_t y_{t+1}^n$  from both sides we get:

$$x_t = E_t x_{t+1} - \phi (i_t - E_t \pi_{t+1}) + g_t. \quad (\text{A.0.34})$$

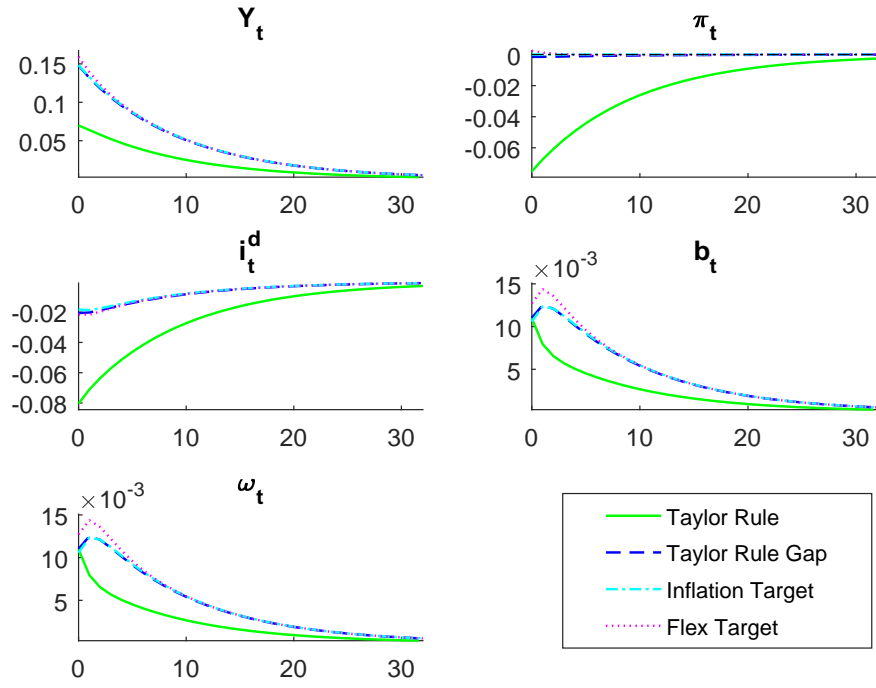
where  $g_t = E_t y_{t+1}^n - y_t^n$ .

The final step is to recognize that the linearized version of (A.0.9) is:

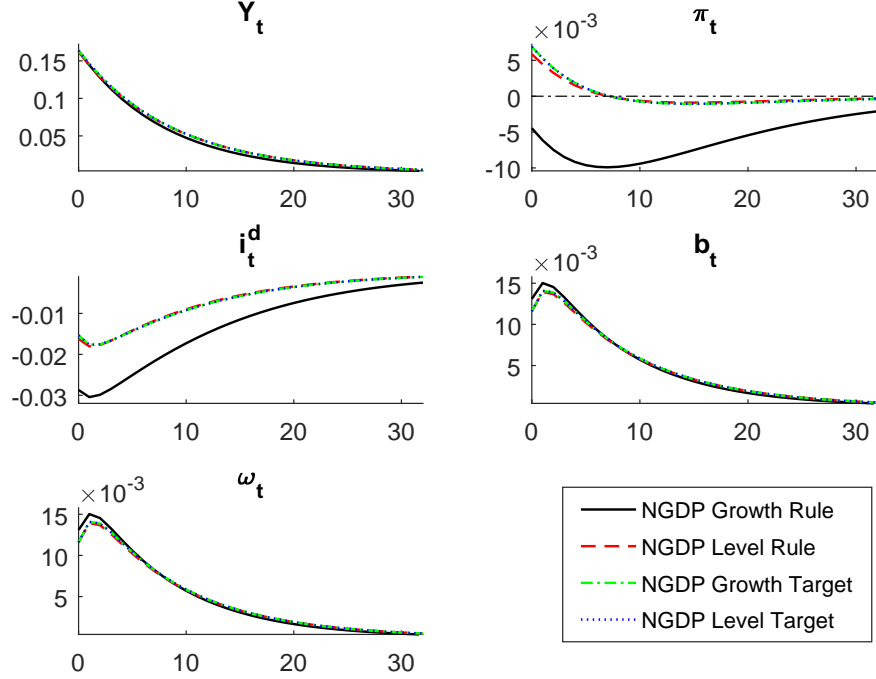
$$m_t - p_t = \frac{\sigma}{\gamma} y_t - \frac{1}{i_{ss}\gamma} \hat{i}_t. \quad (\text{A.0.35})$$

## APPENDIX B

### SUPPLEMENTAL MATERIAL FOR CHAPTER III

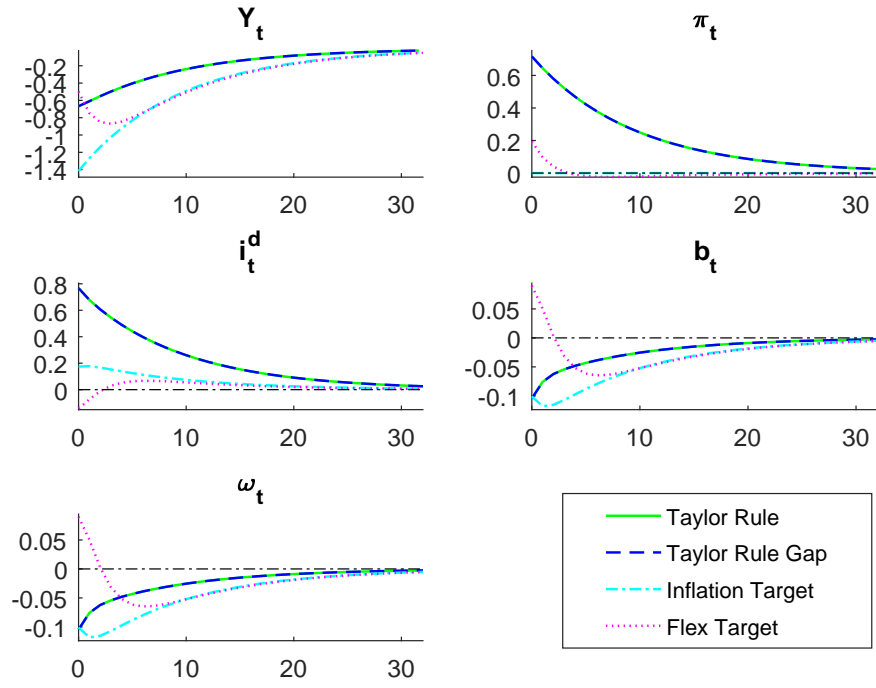


(a) Impulse response function for traditional policies.

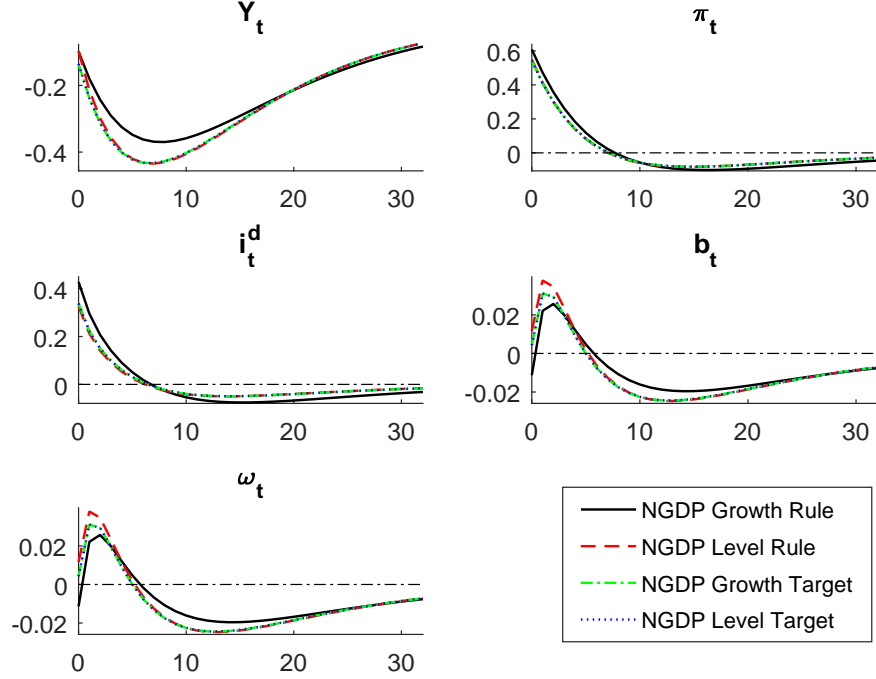


(b) Impulse response function for NGDP based policies.

FIGURE 24. Impulse Response Functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to leisure shocks.

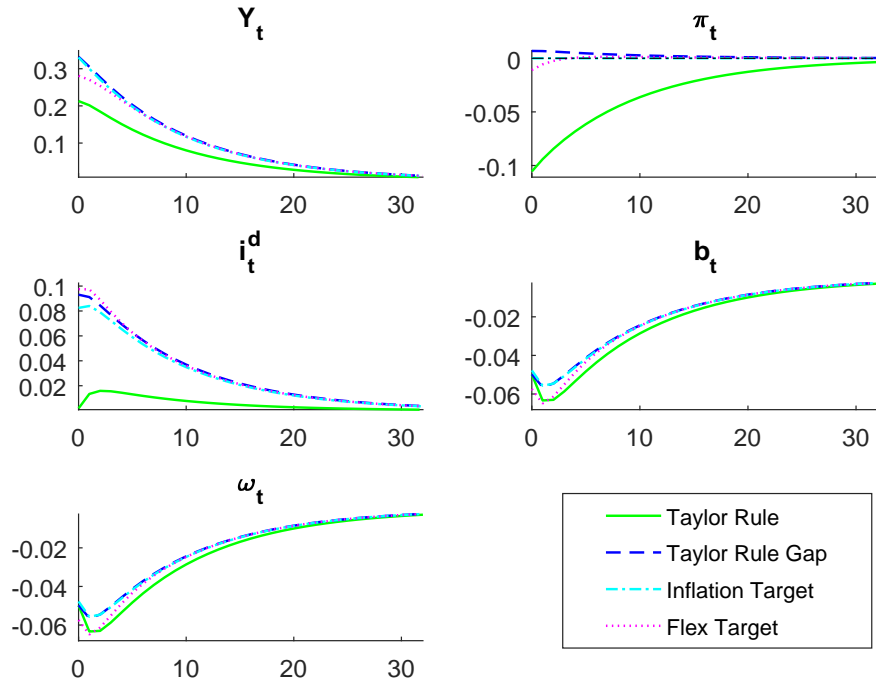


(a) Impulse response function for traditional policies.

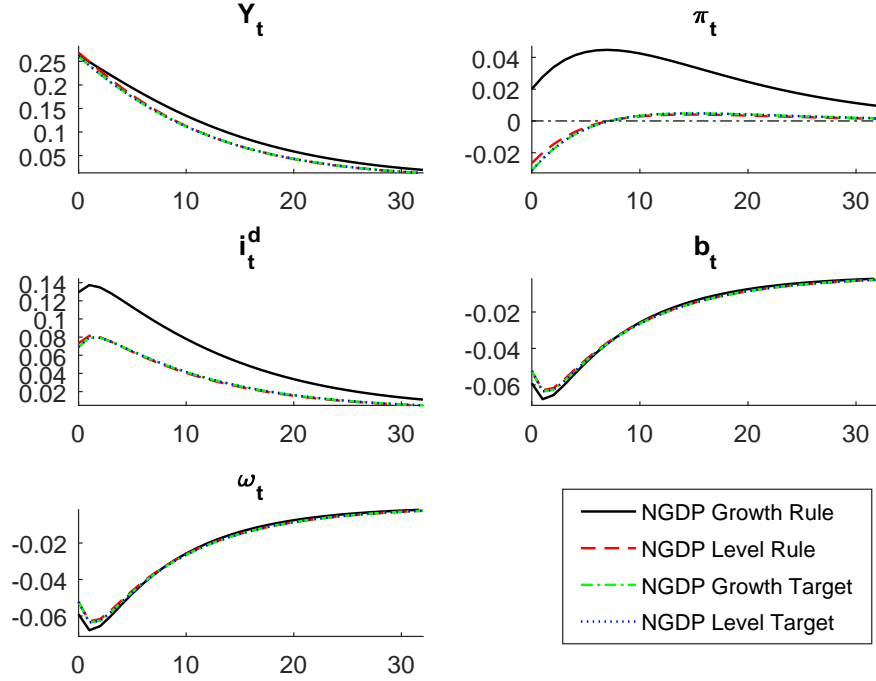


(b) Impulse response function for NGDP based policies.

FIGURE 25. Impulse Response Functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to wage markup shocks.

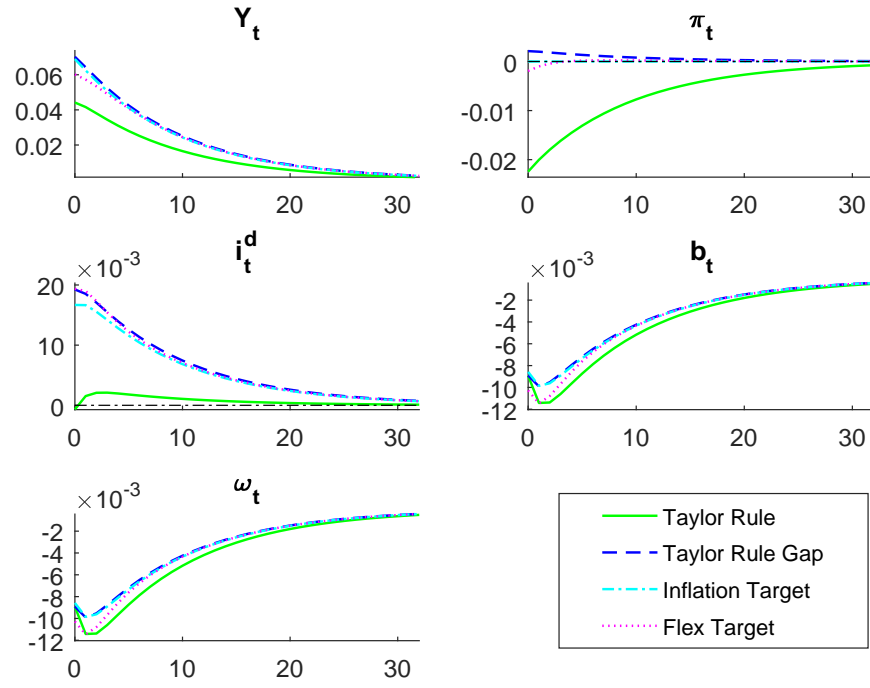


(a) Impulse response function for traditional policies.

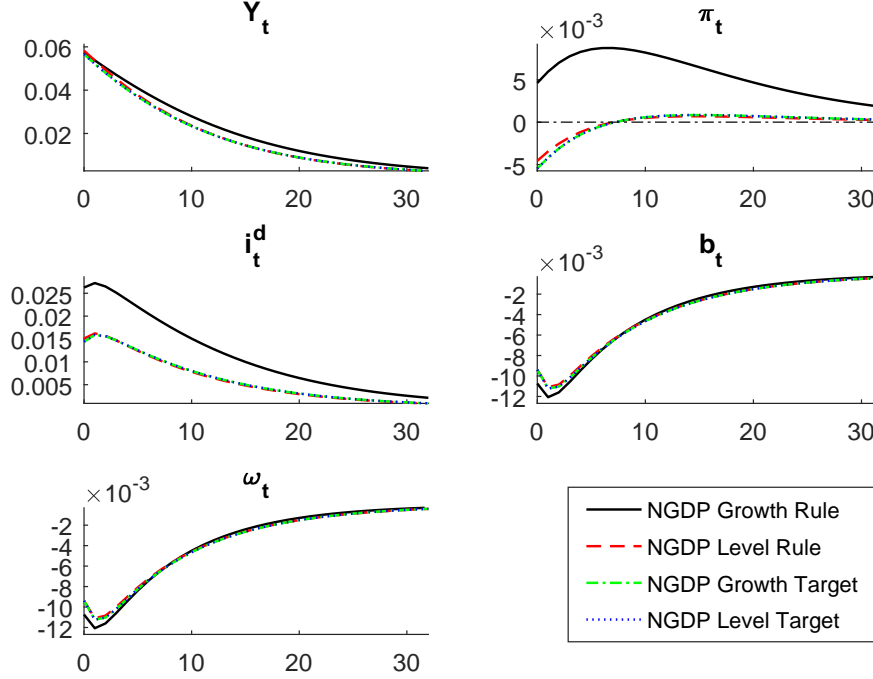


(b) Impulse response function for NGDP based policies.

FIGURE 26. Impulse Response Functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to public spending shocks.



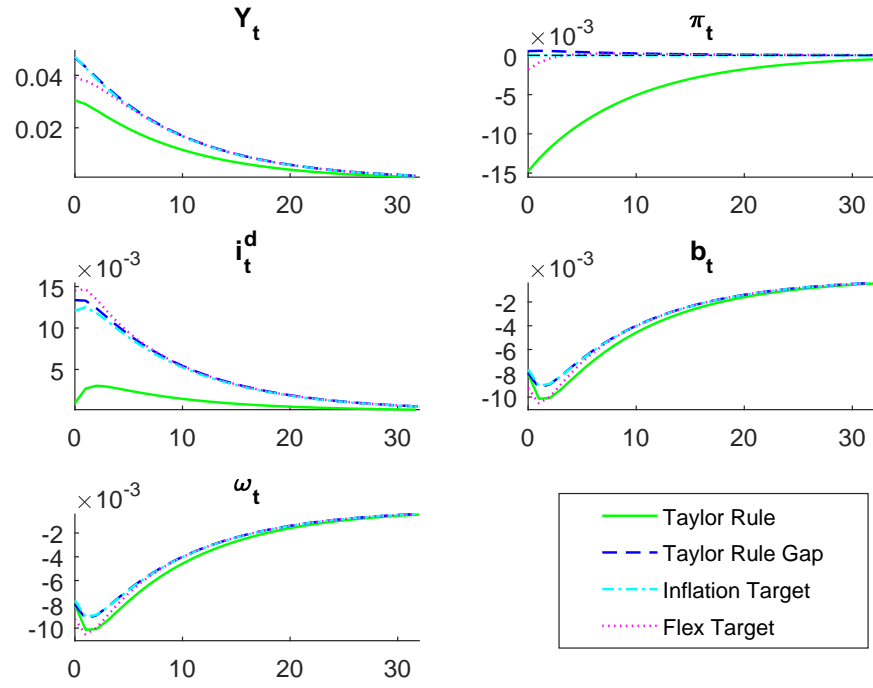
(a) Impulse response function for traditional policies.



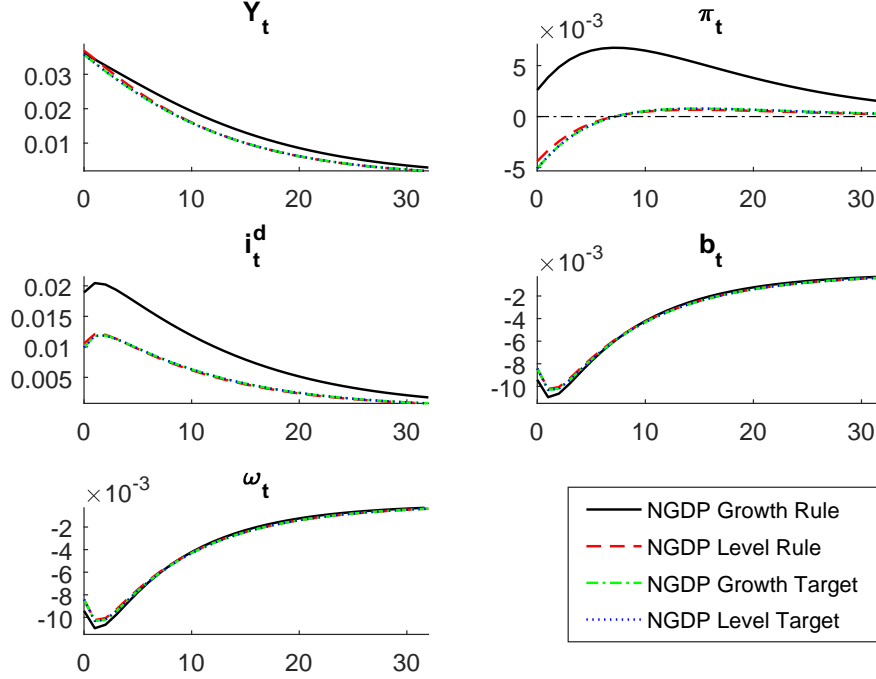
(b) Impulse response function for NGDP based policies.

FIGURE 27. Impulse Response Functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to borrower marginal utility shocks.





(a) Impulse response function for traditional policies.



(b) Impulse response function for NGDP based policies.

FIGURE 28. Impulse Response Functions for  $Y_t$ ,  $\pi_t$ ,  $i_t^d$ ,  $b_t$ , and  $\omega_t$  to saver marginal utility shocks.

## APPENDIX C

### SUPPLEMENTAL MATERIAL FOR CHAPTER IV

This derivation primarily follows the ones that can be found in Ascari and Ropele (2009) and Ascari and Sbordone (2014). Assume that the model is the same as the one found in the Chapter 2 appendix except steady state inflation is non-zero and firms are able index prices to either past or trend inflation. This means that a firm that the price of a firm at period  $t + i$  that reset its price in period  $t$  is:

$$P_{t|t+i} = P_t^* \Omega_{t|t+i-1} \tag{C.0.1}$$

$$\Omega_{t|t+i-1} = \left( \frac{P_{t+i-1}}{P_{t-1}} \right)^{\epsilon\omega} \Pi^{\epsilon(1-\omega)}. \tag{C.0.2}$$

Here  $\Pi$  is gross trend inflation,  $\epsilon$  is the degree of indexation, and  $\omega$  determines the relative weight between past inflation and trend inflation.

The firms' optimization problem is given by:

$$\max_{P_{jt}} \sum_{i=0}^{\infty} E_t^*(\beta\varphi)^i \Lambda_{t|t+i} \left( \frac{P_{jt} \Omega_{t|t+i-1}}{P_{t+i}} Y_{t|t+i} - \frac{W_{t+i}}{P_{t+i}} Y_{t|t+i}^{1/a} \right) \tag{C.0.3}$$

$$\text{s.t. } Y_{t|t+i} = \left( \frac{P_{t|t+i}}{P_{t+i}} \right)^{-\theta} Y_{t+i} \tag{C.0.4}$$

At this point we need to be sure that the value of trend inflation is low enough such that the above summations converge in the deterministic steady state. Ascari and Ropele (2009) shows that this requires that the following holds:

$$\Pi < \min\left\{\left(\frac{1}{\varphi}\right)^{\frac{1}{(\theta-1)(1-\epsilon)}}, \left(\frac{1}{\beta\varphi}\right)^{\frac{1}{\theta(1-\epsilon)}}\right\} \quad (\text{C.0.5})$$

The first order condition to the firm's optimization problem can be written as:

$$\left(\frac{P_t^*}{P_t}\right)^{1+\epsilon\frac{1-a}{a}} = \frac{\theta F_t}{a(\theta-1)G_t} \quad (\text{C.0.6})$$

$$F_t = \sum_{i=0}^{\infty} E_t^*(\beta\varphi)^i \Lambda_{t|t+i} \frac{W_{t+i}}{P_{t+i}} Y_{t+i}^{1/a} \left(\frac{\Omega_{t|t+i-1}}{\Pi_{t|t+i}}\right)^{\frac{-\theta}{a}} \quad (\text{C.0.7})$$

$$G_t = \sum_{i=0}^{\infty} E_t^*(\beta\varphi)^i \Lambda_{t|t+i} \left(\frac{\Omega_{t|t+i-1}}{\Pi_{t|t+i}}\right)^{1-\theta} Y_{t+i}. \quad (\text{C.0.8})$$

Additionally, the price level is evolving according to:

$$P_t[\varphi(P_{t-1}\Omega_{t-1|t-1})^{1-\theta} + (1-\varphi)(P_t^*)^{1-\theta}]^{1/(1-\theta)} \quad (\text{C.0.9})$$

Finally while under zero trend inflation the first order approximation to price dispersion was just its stead state value, now it must be tracked:

$$s_t = (1-\varphi)\left(\frac{P_t^*}{P_t}\right)^{-\theta} + \varphi\left(\frac{\Pi_t}{\Omega_{t-1|t-1}}\right)^{\theta} s_{t-1}. \quad (\text{C.0.10})$$

Where  $s_t \equiv \int_0^1 (P_{i,t}/P_t)^{-\theta} di$ .

You can then log linearize these equations and substitute (C.0.9) into the other two to arrive at the log linearized equations for the Phillips Curve and the evolution of price dispersion presented in Chapter 4. The coefficients in those equations are:

$$\kappa_1 = -\frac{\Pi^{(\theta+1)(-(\epsilon+1))} (\Pi^{\theta\epsilon+1} - \varphi\Pi^{\theta+\epsilon})}{a\varphi(a(\theta-1) - \theta)} (\Pi^{\theta\epsilon+1} (a^2\sigma_c - \quad \quad \quad) \quad \quad \quad) \quad \quad \quad (C.0.11)$$

$$a^2 + \varphi\beta\theta\Pi(\epsilon-1) + \sigma_n(\varphi\beta\theta\Pi(\epsilon-1) + a) + a) - a^2\varphi\beta(\sigma_c-1)\Pi^{\theta+\epsilon}) \quad \quad \quad (C.0.12)$$

$$\kappa_2 = \frac{\sigma_n\Pi^{-\theta-\epsilon} (\varphi\Pi^{\theta+\epsilon} - \Pi^{\theta\epsilon+1}) (\varphi\beta\theta\Pi(\epsilon-1) + a)}{\varphi(a(-\theta) + a + \theta)} \quad \quad \quad (C.0.13)$$

$$\kappa_3 = -\frac{\beta\Pi^{(\theta+1)(-(\epsilon+1))} (-a^2(\theta-1)\Pi^{\theta+\theta\epsilon+\epsilon+1} + a\varphi\theta\Pi^{2(\theta+\epsilon)} - \theta^2(\epsilon-1)\Pi^{\theta\epsilon+2} (\Pi^{\theta\epsilon+1} - \varphi\Pi^{\theta+\epsilon}))}{a(a(\theta-1) - \theta)} \quad \quad \quad (C.0.14)$$

$$\kappa_4 = -\frac{\beta\Pi^{(\theta-1)(\epsilon-1)} (a\Pi^{\theta-\theta\epsilon+\epsilon-1} + \theta\Pi(\epsilon-1)) (\varphi\Pi^{\theta-\theta\epsilon+\epsilon-1} - 1)}{a(\theta-1) - \theta} \quad \quad \quad (C.0.15)$$

$$\xi = \frac{\varphi\theta (\pi^\epsilon - \pi) \pi^{\theta+\theta(-\epsilon)-1}}{\varphi\Pi^{\theta-\theta\epsilon+\epsilon-1} - 1} \quad \quad \quad (C.0.16)$$

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